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MATHEMATICAL ANALYSIS OF SOLID WASTE COLLECTION

David H. Marks, et al

Johns Hopkins University
Baltimore, Maryland

1970

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MATHEMATICAL ANALYSIS
OF SOLID WASTE COLLECTION

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Department of Geography and Environmental Engineering
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FOREWORD

An important objective of the Bureau of Solid Waste Management is to aid in developing economic and efficient solid waste management practices. As authorized under the Solid Waste Disposal Act (Public Law 89-272), the Bureau has made almost 100 research grants to nonprofit institutions in this effort to stimulate and accelerate the development of new or improved ways for handling the Nation's discarded solids. The present document, which is reproduced as received from the grantee, reports on work completed under one of these research grants.

The general problems were investigated by the grantee in an analytical framework. These are the location of transfer stations and the routing of collection vehicles. Mathematical optimization models for these problems were constructed, using existing or slightly modified algorithms. These models, particularly the ones for transfer facility site selection, may be used in choosing the economically optimal plan from among a large number of alternatives. This information serves as nonquantifiable information in making the final choice.

Several of the models were tested by the grantee using data taken from the City of Baltimore. Feasibility of transfer stations, the desirability of rail haul, and the cost of increased collection frequency were investigated in the testing process. These applications serve to demonstrate the final usefulness of the models.

--RICHARD D. VAUGHAN, *Director*
Bureau of Solid Waste Management

ABSTRACT

The application of operations research to the analysis of solid waste collection systems is studied. Models and techniques for facility location and routing are discussed and extended. Three different types of problem areas are viewed. Chapter II covers facility location, in particular the location of transfer facilities within a large-scale system. Chapter III deals with finding optimal flow through given systems with added constraints. In particular, a multi-commodity truck assignment problem where a common vehicle fleet is used to carry several commodities between supply and demand points is extended and solved. Chapter IV is concerned with vehicle scheduling problems where routes are to be found for individual collection vehicles among various tasks. In Chapter V, the analysis of an actual large-scale solid waste collection system, that of Baltimore, Maryland, is carried out using some of the methods developed. Results of the analysis indicate that the use of such models in the study of large-scale public systems can supply a great deal of information about, and insight into, the management and planning of the system.

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CHAPTER I. OPERATIONS RESEARCH AND SOLID WASTE COLLECTION

Introduction

This thesis is concerned with the development of analytical techniques to aid in the management and planning of solid waste collection systems. Specifically, the techniques to be presented are operations research models of routing and facility location problems that are an inherent part of any collection or delivery system. The models are most applicable in the private sector of the economy, since they seek to minimize a well-defined and quantifiable cost subject to explicit constraints. Solid waste collection systems, however, are for the most part functions of the public sector rather than the private sector. There is a divergence between the objectives of a public and private system, with the public objective function being more vague and difficult to express formally. Constraints on the public system, especially those of a political or social nature, may be difficult to measure, and criteria of effectiveness may not exist in units which are commensurate.

All this does not mean that operations research models and tools should not be applied to public sector problems such as solid waste collection system management. As with private sector models, analysis is performed as an aid to the decision maker and not as a replacement for him. Careful use, based on an appreciation of the factors that the models cannot consider explicitly, will allow meaningful analysis to be carried out. The amount of analysis attempted in such large scale systems until recently has been limited by the sheer size and complexity of the problems. Now with the availability of operations research techniques and the aid of high speed computers, the potential to increase greatly the understanding of such systems is apparent. It is hoped that this will be an essay both in the advantages to be gained by systematic investigation and the pitfalls involved in blindly following models as a substitute for judgment and intuition.

The purpose of this work is twofold. First, the routing and facility location problems of a collection system will be analyzed and specific techniques suggested for approaching their solution. Second, the application of these techniques to the investigation of a large scale solid waste collection system as an indication of the questions amenable to analysis by these models will be presented.

A. The Scope of the Solid Waste Problem

The solid waste system is defined to include the generation of waste at a source, its collection and transport and the disposal methods available. It is a large, expensive and vital on-going system with peculiarities and problems that are in urgent need of analysis. The yearly cost of operating the system in the United States is reported by Black (1964) to be 3 billion dollars per year. This amount makes it one of the foremost public works expenditures, ranking after education, highways, welfare, and fire and police among local government expenditures. The amount of waste generated in the United States from all sources approaches 125 million tons per year. With increasing indications that these costs and volumes will continue to rise drastically (Ludwig and Black, 1968), it seems that detailed analysis of the system would be in great demand.

The main difficulty in analysis is determining some measure of effectiveness for the service provided. The evolution through the years of solid waste management from a private to a public problem is indicative that the service provided is a public good. The entire community suffers from the viewpoint of public health and aesthetics when even one of its members refuses to dispose properly of his solid wastes. The cost of enforcing sanitation ordinances within a private system may far outstrip the cost of a publicly owned system. A measure of efficiency of a public system might be the degree of service offered to the customer in terms of frequency of collection, types of wastes removed, locations from which waste is collected, and the general level of satisfaction shown by the consumers in order to provide the incentive for all to dispose properly of waste material.

Management to achieve some desired level of service at minimum cost is the goal, then, of the system operator, and he must study the various control alternatives available to improve the efficiency of the system. Cost analyses by Ludwig and Black (1968) reveal that 85 percent of the solid waste system cost is due to collection and only 15 percent to disposal. This does not necessarily mean that the only approach to the problem is through improvements in collection. In spite of the fact that most analysts consider that waste loads are given and begin designing a system from that point, it would seem that collection costs could be decreased by eliminating wastes before they require collection and transport. From a technologic point of view, materials which are difficult or expensive to handle or dispose of might be replaced by those which are not. The impetus for such a move could be either legislative, through laws preventing the use of objectionable material, or economic, through special taxes which would make using the material economically infeasible. Economic incentives might also change current waste material into production inputs. Education of the public might also reduce waste load produced. Technologic advance in the form of better in-home disposal units is a hope for the future even though present devices are less than adequate. Present day incinerators create an air pollution hazard and still have an ash disposal problem. Garbage grinders simply trade a solid waste collection system for an underground liquid waste system, and disposal problems continue.

Some ideas have already been advanced for improving collection. Prominent among these is the building of transfer facilities within a city at which the special purpose collection vehicles could discharge their loads to special purpose transport vehicles and thus return to their collection jobs sooner. The suggested transport vehicles have ranged from large tractor trailers to barges and railroad cars. Recommendations have also been heard for even more specialized collection vehicles such as small carts hauled in tandem by a jeep or motor scooter to a rendezvous with a larger transport vehicle. Zandi (1968) has proposed an extensive pipe line system for moving solid wastes under pressure as an alternative to surface collection. All of these control methods require some amount of investigation and testing. Yet, the most important thing to realize about the solid waste system is that it is too big, complex, and vital to allow actual experimentation without great expense or the potential of great chaos. Coupled with this are all the other problems involved with studying large scale public systems. A relevant data base is probably non-existent. The political implications of who controls the system and who pays for which service may be the most persuasive argument for throwing out a scheme that might otherwise seem quite efficient. Since large investment has already been made in attempting to manage the system, the designer is denied the luxury of starting from the beginning and is often saddled with working around the blunders of the past. To this morass the analyst hopes to bring some order.

Since collection makes up 85 percent of the cost of the solid waste management system, it is to this phase that the attention of this work will be directed. Successful analysis of the collection process could result not only in means for making it run more efficiently, but also provide a means for measuring how non-collection alternatives which reduce waste load must be priced in order to make them better choices than the current approach to the problem.

B. Literature Review of Operations Research in Solid Waste Collection

A search of the pertinent literature reveals that very few authors have concerned themselves with the application of operations research to the study of solid waste collection systems. The work that does appear falls into two categories: broad scale attempts at outlining the nature and interactions of the entire waste management system, in which collection is but one part, and narrower, more specific investigations of the collection and disposal operations suited to the detailed study of present operating systems. The most complete of the broad scale studies is one still under way at the University of California at Berkeley entitled "Comprehensive Studies of Solid Waste Management". In the first annual report of the study (Golueke and McGauhey, 1967) one of the stated objectives of the study is "to explore the potential of operations research to help in the definition and solution of the solid waste disposal problem". Basically, two problems were investigated. The first was to find a means for evaluating solutions for the total refuse disposal problem that would not only be applicable to a wide spectrum of alternatives but would be sensitive to environmental and governmental constraints as well. This involved the development of conceptual models which gave insight into the operation of the system. As a second problem, the study undertook to build mathematical models for the conceptual models. These included a waste generation model, a waste collection, treatment and disposal model, a regional economic model, and models for population, public health aspects, land use and process technology. The most interesting of these models from the viewpoint of operations research techniques was the waste collection, treatment and disposal model. The subgroup working with this problem looked at the means of determining how solid wastes should move from sources through treatment and processing to disposal sinks. They identified the problem as one of network analysis and noted that solution should be gained by a graph theoretic approach. The outcome of the development of specific techniques for solving the model as formulated is reported by Anderson (1968), who was one of the study group. Anderson was interested in modeling how the flow would be optimized through a given system which contained existing facilities and potential new facilities. He identified the problem as a study of a trans-shipment network with some peculiarities due to the nature of the processes involved. In considering a total waste management model including intermediate treatment facilities, it is necessary to consider the changes in volumes and in the state of the wastes which is not possible using ordinary network algorithms. He suggests two solution techniques: if the number of disposal sinks is small, a branch and bound model may be used to generate optimal flow through the system. If not, he also presents an out-of-kilter algorithm for solution which includes an added feature of allowing flow through a node to be proportioned in a fixed ratio. In both models all costs are linear, and the system under investigation considers the gross shipment between points and not the routing of individual vehicles among small collection tasks. Also, Anderson dealt with the problem of finding flows through a known or given configuration. To consider various combinations of new facilities, each scheme must be evaluated separately. No methodology is presented for a systematic search over facility location possibilities. The assumption of linear costs omits some of the important aspects of the problem. Although nonlinear costs may be given linear approximations, this is only valid if the nonlinear costs are convex. Facilities costs are commonly concave or quasi-concave (fixed charge). Thus linear approximation either will not be successful or must be done in such a manner that one loses confidence in the meaning of the results of the solution.

Other studies defining the solid waste system have been carried out by consulting organizations for governmental units, but few if any have gone beyond the characterization of the system and attempts at simulation of the process. Such studies include reports by Management Technology, Inc. (Anon. 1966b) for the State of Maryland, and by Aerojet General (Anon. 1965) for the State of California.

The first work in attempting to understand the complicated microstructure of the detailed collection process at the individual block and vehicle level has been by simulation models. Quon, Charnes and Wersan (1965) presented a model of a collection system with particular attention to queuing problems at the disposal point, using data from Winnetka, Illinois. Truitt, Liebman and Kruse (1968, 1969) constructed a simulation model in which the effect of a proposed transfer station may be investigated. Their results are based on data from Baltimore, Maryland. Quon, Tanaka and Wersan (1969) continue their earlier simulation work with a model for studying changes in work rules and collection policy.

Other authors have dealt with different aspects of the collection problem that are more amenable to analytical attack. Coyle and Martin (1967) presented a heuristic method based on dynamic programming for aggregating small collection areas into work schedule assignments for crews and vehicles. Skelly (1968) presented a fixed charge model for looking at the large scale problem of transportation of wastes in which the variables include the alternative location of transfer stations. He used for a solution technique a heuristic fixed charge algorithm developed by Walker (1968). Skelly's work is discussed in greater detail in Chapter II. Wolfe and Zinn (1967) have presented a simple model for an overall crude evaluation of a large system as an example of systems application in public works problems. A small trial problem was solved using linear programming.

C. The Analysis of Solid Waste Collection

The analyst involved with a solid waste collection system is concerned with answering a set of questions about the system so that he can better understand it. This understanding will lead to improved decision making. The sort of questions he might ask are the following:

1. What are the goals of the system? What frequency of collection and types of service should be offered by the system? How will changing the service affect cost?
2. What types of vehicles should be used, and how many?
3. How many personnel are needed and what should their duties and work rules be?
4. What route should be assigned to each vehicle? How should the city be divided into administrative subgroups?
5. Are there parameters of the system to which system costs and variables are particularly sensitive?
6. If there is additional money available for research, into what aspect of the system should further study be encouraged?
7. Should there be intermediate transfer stations for the deployment of wastes to more specialized transport vehicles? Where should they be located and what type of equipment should they contain?
8. What type of transport vehicle would be used in transfer from a transfer station to final disposal?
9. What type of disposal alternative should be chosen and where should it be located?
10. What would be the effect on the system of new technology in in-house waste reduction? In new disposal technology?
11. How will the stochastic nature of waste generation affect the analysis? How will the solution change as the area to be served continues to grow and spread?

12. What are the effects of political, social and economic constraints? How much should be spent on aesthetic factors? Is regional grouping a feasible alternative?

Ideally, to answer all these questions, the analyst would like to build a model that would be able to follow the collection process in its minutest detail and be able to manipulate all of the myriad parameters that could possibly affect the solution. However, its magnitude and complexity are such that to consider the system in its entirety and encompass every possible detail is quite difficult, if not impossible. For this reason, simplifying assumptions will be made in model development which do not allow the detail mentioned but will allow some approximation of the problem. Basically, two different approaches are taken. If one is willing to neglect the problems of routing of the individual vehicles through their collection tasks, the large scale problem of how material should move through the system and how transfer facility location may be chosen can be approached optimally. This will be called the "macro-scale" or "flow of materials" subproblem. If the location of facilities and the collection areas assigned are known for a set of collection vehicles, the small scale problems of routing vehicles among individual collection tasks may be approached. This second subproblem will be called the "micro-scale" problem. Extensive discussion and development of both of these problems will be presented. Application of the proposed models to the investigation of a solid waste collection system will be emphasized.

The general outline of this thesis is as follows: Chapter II is concerned with the location of transfer facilities. If the problem is treated as a large scale "flow of materials" problem, it is found to belong to a class of problems known in the literature as location problems, warehousing problems and plant location problems. Techniques for solving these problems are discussed and a special method is developed that is particularly adaptable to the solid waste collection problem.

Chapter III also deals with the large scale flow of goods problem. In this case, the network structure of the system is given and optimal flow through it is found for more extensive constraints than were allowed in Chapter II. In particular, multicommodity flow using a common vehicle fleet is studied. Extension of a multicommodity truck assignment problem by Szwarc (1967) is presented and computational experience is discussed.

Chapter IV deals with the micro-scale routing problems. These are found to be represented by several classes of problems in operations research. The most general problems discussed are known as vehicle scheduling problems and traveling salesman problems. An original algorithm for the m-salesman traveling salesman problem is presented. Problems of routing in street networks are shown in some cases to be chinese postman problems, and this problem is discussed.

Chapter V deals with the analysis of an actual system using some of the techniques developed in the thesis with fairly crude data.

Chapter VI presents a summary, and some conclusions and suggestions for further research.

CHAPTER II. THE LOCATION OF TRANSFER FACILITIES

A. Problem Description and Literature Review

Determining the location of intermediate facilities where transfer of solid waste may take place from specialized collection vehicles to vehicles more suited for long-haul transportation is a problem that has received considerable attention within the past few years. The fundamental questions involve the desirability of transfer stations, their number, location, and capacity, and the specific functions which they should perform. These decisions may be viewed as a trade-off between the building of facilities and the cost of transportation. Two points are immediately clear. First, this is a general problem of facilities location in both the private and public sector rather than a problem specific to solid wastes, and studies in the general area of facilities planning may be applicable. Second, assuming that facility costs and transportation costs can be defined, it may be possible to derive some mathematical formulation of the problem.

A search of the operations research and economics literature reveals that a considerable interest has been generated and mathematical formulation for certain aspects of the problem has been developed. The methods developed have been for the large scale "flow of goods" features of the problem in which the concern has been with selecting paths over which materials flow from collection areas to facilities for transfer and disposal. The routing of vehicles within collection areas is usually ignored, as it would add a complexity to the problem that would make it intractable. However, since the goal of model formulation is to obtain some structure that may be solved and manipulated in order to gain better understanding of the system as a whole, such work is still of great importance. Care should be taken, though, in the use of the solutions generated. An optimal answer to a subproblem may have little meaning in the context of the larger problem, even though manipulation of the subproblem to show the sensitivity of its optimal solution to changes in parameters and control methods may provide a great deal of insight and information for the analyst.

The problem area as discussed in the general literature is known by several names: the location-allocation problem, the plant location problem, the central facilities problem and the warehousing problem. To a large extent, all of these names imply the same general problems, with differences depending on the assumptions made in order to facilitate modeling and solution techniques for the models. As noted, the general formulation is based on trading off facility cost versus transportation cost. The problem is to find that system configuration that minimizes the sum of the two costs.

Analysts who have worked on this problem have been faced with two basic problems in definition and assumptions which have led to two different structures for analytical approach. The first assumption is whether demand is spread, uniformly or otherwise, continuously across the entire area to be served, or is located at discrete points. The second assumption is whether potential facilities sites are infinite in number or are limited to a discrete number of feasible alternatives. If the assumption is made that both demand and facilities locations are infinite the analysis procedure is quite ill-defined and little analytical success has been obtained in such problem solutions. However, once some form of limiting assumption is made of the solution space, considerable progress in solution technique has been found. It should be pointed out that such discrete assumptions are not severe from an engineering point of view. In fact, within a city there are not an infinite number of sites for solid waste facilities, but a finite number based on available land, zoning and concentrations of population and the location of existing structures. Demand, too, does tend to cluster and an assumption dividing a region into areas with each represented by its demand at some centroid is not unreasonable in most cases. The two approaches may be structurally distinguished by regarding the problem with infinite solution space for demand and facility location as location on a plane, and the discrete problem as location on a network.

The problem names most associated with location on a network are the warehousing problem and the plant location problem. In the warehousing problem, warehouses or central facilities are built to serve as supply points for satisfying demand in the region and the flow of goods takes place from the facility to the demand area. In the plant location problem, the central facility is a production facility which requires a flow of goods to it from the surrounding regions. From an analysis point of view, the problems are identical and the problem names may be used interchangeably. Thus the following description of a warehouse problem may be converted to a description of the plant location problem by changing the direction of flow of goods.

The statement of the warehouse problem is: Given a number of demand areas for a certain product, each with a demand D_j, and a number of alternative sites where facilities may be built to satisfy these demands, determine where facilities should be built and which demand areas are to be served by each facility. The objective is that the sum of the transportation cost and the amortized facility cost is minimized.

The general mathematical formulation is:

Minimize: $$\sum_{j=1}^{n} \sum_{i=1}^{m} d_{ij}(x_{ij}) + \sum_{i=1}^{m} F_i(y_i) \qquad (2\text{-}1)$$

subject to: $$\sum_{j=1}^{n} x_{ij} = y_i \qquad i=1, 2, \ldots, m \qquad (2\text{-}2)$$

$$\sum_{i=1}^{m} x_{ij} = D_j \qquad j=1, 2, \ldots, n \qquad (2\text{-}3)$$

$$x_{ij} \geq 0 \qquad \begin{array}{l} i=1, 2, \ldots, m \\ j=1, 2, \ldots, n \end{array} \qquad (2\text{-}4)$$

$$y_i \geq 0 \qquad i=1, 2, \ldots, m \qquad (2\text{-}5)$$

where: x_{ij} = amount shipped from warehouse i to demand area j (a decision variable)

y_i = total amount shipped from warehouse i (a decision variable)

$d_{ij}(x_{ij})$ = cost of shipping x_{ij} from i to j (dollars)

$F_i(y_i)$ = cost of building and operating warehouse i when y_i is to be shipped from it (dollars)

D_j = demand at area j

n = number of demand areas

m = number of proposed warehouse sites.

If the functions $F_i(y_i)$ and $d_{ij}(x_{ij})$ are linear, the problem becomes a simple transportation problem. Unfortunately, the function $F_i(y_i)$ is frequently nonlinear, and generally exhibits a large fixed investment for land, foundations, utilities, etc., before any amount may be manufactured or shipped. Once the facility is begun, the marginal cost of another unit of storage or manufacture may decrease (economies of scale). Such a function is termed concave and is not amenable to linear programming. This is the feature which has drawn the most attention from researchers in this field, and the method of treating the concave cost function is the key element of most procedures.

Among the earliest work in the literature is a paper by Baumol and Wolf (1958), in which the authors approach the problem by using a linear facility cost instead of a fixed charge. Then the problem may be formulated as a transportation problem. The resulting solution is then adjusted, using a heuristic procedure, to reflect the fixed charge aspects of the problem. This discussion will be limited to five fairly recent approaches, all thought to have some application and validity. The papers by Feldman, Lehrer and Ray (1966) and by Kuehn and Hamburger (1963) represent two early formulations of the problem which rely on heuristics to achieve good solutions. Balinski (1965) formulated the problem as an integer programming problem which is capable of solution for small cases. Efroymson and Ray (1966), Spielberg (1969) and the author of this thesis propose methods which achieve optimal solutions to larger mathematical problems as structured through branch and bound techniques.

As noted before, Kuehn and Hamburger used a heuristic algorithm to find good solutions to the warehousing problem. They assumed that transportation costs are linear functions of the amount transported and that facility costs are of the form,

$$F_i(y_i) = a_i + b_i y_i \quad \text{if the facility exists}$$
$$= 0 \quad \text{if it does not}$$

Thus $F_i(y_i)$ consists of a fixed charge that is independent of the storage and a linear cost which does depend on storage, if the facility exists.

The method starts with one facility and tries adding another facility to see if the total cost can be decreased. The general assumption is that the best N facilities contain the set of the best N-1 facilities. Termination of the procedure occurs when it appears that another facility cannot be added without increasing the total cost.

One difficulty in working with heuristics is that there is no way to tell how far the final answer obtained is from the true optimal solution. This difficulty tends to decrease the validity of an analysis of sensitivity by obscuring the cause of a change in the solution which might occur when a parameter is altered. An improvement could be due to the parameter change or to luck in the computations. Thus, extreme care should be taken in using a heuristic for sensitivity analysis.

Feldman, Lehrer and Ray assumed a more general form for the facility cost, that of a continuous concave function. They also assumed that transport cost is linear in the amount shipped between points. Since $F_i(y_i)$ does not have a linear variable cost but a variable cost that is dependent on how many demand areas are being served by the facility, the problem of assigning demand areas to existing facilities is not as easy as before. An approximation method is used to accomplish this assignment. Whereas Kuehn and Hamburger began with one facility, Feldman et al. started with all potential facilities existing and dropped some out. Termination occurs when no further savings can be obtained by dropping facilities. Balinski (1964, 1965) developed a formulation for the plant location problem that could be solved by integer programming techniques. In order to do this, $F_i(y_i)$ is specified as a single fixed charge, F_i, if the facility exists. Capacity constraints are not specified for the facilities. The formulation is as follows:

$$\text{Minimize:} \quad \sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij} x_{ij} + \sum_{i=1}^{m} F_i y_i \qquad (2\text{-}6)$$

subject to: $$\sum_{i=1}^{m} x_{ij} = 1 \qquad j=1, 2, \ldots, n \qquad (2\text{-}7)$$

$$0 \leq x_{ij} \leq y_i \leq 1 \qquad i=1, 2, \ldots, m; \; j=1, 2, \ldots, n \qquad (2\text{-}8)$$

$$y_i = (0, 1) \qquad i=1, 2, \ldots, m \qquad (2\text{-}9)$$

where: $c_{ij} = t_{ij} * D_j$

t_{ij} = the unit transportation cost from facility i to customer j

D_j = demand at j

x_{ij} = the fraction of D_j supplied from facility i

F_i = the fixed charge associated with facility i

n = the number of customers

m = the number of proposed facilities

Equation (2-7) specifies that each customer must have all his demands satisfied and inequality (2-8) requires that demand may not be satisfied by a facility unless it exists. Note that even for a small problem the above formulation has a formidable number of constraints and variables. For an N-customer, M-plant problem there are N x M + N + M constraints and N x M + M variables. Thus, for the case M=10 and N=30, the problem to be solved has 340 constraints and 310 variables.

It would seem, at first glance, that if the integer requirement (2-9) were replaced by the simple constraint $y_i \leq 1$, the solution to the resulting linear programming problem would be integer. However, a counter example is reported and the means of solution first attempted is an integer programming cutting plane technique of Gomory (1958). This is shown to have a very slow convergence even for small problems, and is abandoned for larger problems. Balinski suggests another scheme for solution which involves partitioning the problem using a method of Benders (1962). This requires the successive solution of more complex integer programming problems. A proof is supplied that the method will converge to the optimal solution in a finite number of steps, but it appears that computation time will be quite large for this type of solution as well.

Efroymson and Ray (1966) presented a method of solution to the same problem using a branch and bound scheme that has been tested computationally with good results. Their formulation of the problem is:

Minimize:
$$\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{i=1}^{m} F_i\, y_i \qquad (2\text{-}10)$$

subject to:
$$\sum_{i=1}^{m} x_{ij} = 1 \qquad j=1, 2, \ldots, n \qquad (2\text{-}11)$$

$$\sum_{j=1}^{n} x_{ij} \leq n_i y_i \qquad i=1, 2, \ldots, m \qquad (2\text{-}12)$$

$$x_{ij},\ y_i = (0, 1) \qquad (2\text{-}13)$$

where:

c_{ij} = cost of supplying the entire demand area j from warehouse i

F_i = fixed charge for establishing warehouse i

y_i = 1 if the i'th warehouse is built, 0 if not

x_{ij} = 1 if the demand area j is served by warehouse i, 0 if not

n_i = maximum number of demand areas that may be assigned to facility i

m = number of possible warehouse sites

n = number of demand areas

An examination of the constraint set shows that Balinski's explicit constraints on the value for each allocation to a facility (2-8) have been collapsed into a smaller set of constraints on the maximum amount that may be assigned to a facility (2-12). This constraint specifies that if the i'th facility does not exist (i.e. y_i=0) no assignments may be made to it. However, if it does exist, assignments may be made to it up to the value of n_i. The value of n_i becomes an important point in the development of the algorithm. The solution method involves an implicit search of all the possible feasible solutions. Suppose all the feasible solutions were represented as branches of a tree with each branch exhibiting a specific value, zero or one, for each variable. In order to avoid searching the entire tree to find the optimal solution, means for estimating the lower bound on the solution by continuing down the tree are available. If at any point on a branch the estimate of the lower bound is greater than a known feasible solution, that branch is dominated and may be abandoned. The speed with which such a procedure progresses to an optimal solution depends on the method used to obtain lower bounds and the rules used for determining which branch to investigate next.

In establishing their branch and bound scheme to solve the problem, Efroymson and Ray had to specify a means for finding a lower bound at each node on a branch. They specified that this could be accomplished by solving the problem denoted by the objective function (2-10) plus constraints (2-11) and (2-12), while ignoring the integer requirement (2-13). If n_i is a capacity constraint such that it is limiting, i.e. areas that might assign to i are excluded because there is not enough capacity at i, the problem to be solved at each node requires the solution of a transportation problem. However, if n_i is greater than or equal to all the areas that could possibly assign to i, the lower bound problem at each node may be solved by a very simple iterative procedure that is very fast. Simply assign the area to the open facility such that transportation cost plus expansion cost at the site is minimized. In order to speed their algorithm, the authors abandoned the requirement for a capacity constraint on facilities by setting n_i at a number greater than or equal to the total number of demand areas that could possibly assign to that facility. Solution of problems with 50 possible warehouse locations and 200 demand areas in 10 minutes on the IBM 7094 is reported.

Spielberg (1969), in a more recent paper, discussed largely the same problem. His algorithm for solution is also branch and bound, but contains many added features which shorten computation time. A great deal of computational experience is reported with problems as large as 100 facilities and 150 demand areas. The author also suggested extensions of the problem, but reported no computational experience for these.

Looking at the special problem of the location of transfer facilities for solid wastes, it is noted that there is an added dimension beyond that of the warehousing formulation. The facilities to be located are intermediate points within a trans-shipment network between the sources and sinks of flow. Thus the transportation cost involves two elements, transportation from source to intermediate point, and from intermediate point to the sink. There are possible facility costs at the intermediate point as well as variable costs at the sources and sinks.

One of the weaknesses of all the previous work in warehousing problems is that there may be no capacity restrictions on the amount of flow through a central facility when in fact the cost estimates and the physical reality of the problem require that there be such a constraint. The problem of locating an intermediate facility with capacity constraints on flow will henceforth be called the capacitated trans-shipment facility location problem. The only previous work on this problem is that of Skelly (1968). In addition to recognizing the above features of the problem, Skelly was also interested in time-staging of construction of facilities and disposal points. He formulated the problem with a series of linear constraints which define the trans-shipment and time requirements. This leaves a large fixed charge integer programming problem.

Skelly chose as his solution method an algorithm by Walker (1968) for the fixed charge problem. This algorithm is simplex-based, i.e. it requires that a basis the size of the constraint set be established and manipulated. Furthermore, it is a heuristic and may not yield a global optimum. The fact that the Walker algorithm is simplex-based leads to some problems in computational efficiency. Trans-shipment problems belong to a class of network flow problems as described by Ford and Fulkerson (1962), which may be written in linear programming form but because of the sparseness and special structure of the constraint matrix, can be solved much more quickly and efficiently by primal-dual labeling procedures. Because of the simplex approach, the size of the problem that Skelly may consider is severely limited. Furthermore, since the value of such models is in the repeated solution using different parameters to show sensitivity, the use of a heuristic algorithm may be self-defeating.

B. The Capacitated Trans-Shipment Facility Location Problem

The remainder of this chapter is devoted to a new solution method for the capacitated trans-shipment facility location problem where there are fixed charges associated with the intermediate facilities. The algorithm guarantees an optimal solution and will solve fairly large problems within moderate computation time. The problem to be solved is stated as follows:

There is a set, K, of sources for a product with an amount S_k at each source. In addition, there is a set of sinks, J, for the product, each with an upper and lower bound on demand of D_j^u and D_j^l. A set of proposed intermediate facility sites, I, has been suggested as trans-shipment points between the sources and sinks. Each proposed facility has a fixed charge, F_i, a variable unit cost, V_i, which is a linear function of the amount shipped through the facility, and a capacity, Q_i. The problem is to find which facilities should be built and which sources and sinks each facility serves, so that the total cost of facilities and trans-shipment is minimized.

In the context of a solid waste collection system, the flow through the system is solid waste material with the sources representing the locations of waste generation and the sinks representing the disposal methods and locations. A lower bound on the amount of solid waste reaching a sink is motivated by the fact that for some types of present disposal processes such as incineration, a minimum throughput is necessary to keep the process operating. Future development in waste disposal technology may also lead to other processes which require continuous operation. The intermediate facilities are transfer stations where transfer may take place from smaller collection vehicles to some other means of transport to the disposal points (sinks). The means of transport from the transfer station may be by vehicles such as large trucks or trains, or by non-vehicle methods such as pipelines. In addition to simple transfer, the activity which takes place at a transfer station may include some form of processing such as sorting, compacting and/or incineration.

In mathematical form this problem becomes the following:

Problem One

Minimize: $$\sum_{i=1}^{m} F_i y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} c^{*}_{ij} x^{*}_{ij} + \sum_{i=1}^{m} \sum_{k=1}^{p} c^{**}_{ki} x^{**}_{ki} \qquad (2\text{-}14)$$

subject to the constraints:

$$\sum_{i=1}^{m} x^{**}_{ki} \geq S_k \qquad k=1, 2, \ldots, p \qquad (2\text{-}15)$$

$$\sum_{j=1}^{n} x^{*}_{ij} = \sum_{k=1}^{p} x^{**}_{ki} \qquad i=1, 2, \ldots, m \qquad (2\text{-}16)$$

$$\sum_{k=1}^{p} x^{**}_{ki} \leq Q_i y_i \qquad i=1, 2, \ldots, m \qquad (2\text{-}17)$$

$$D_j^{u} \geq \sum_{i=1}^{m} x^{*}_{ij} \geq D_j^{l} \qquad j=1, 2, \ldots, n \qquad (2\text{-}18)$$

x^{*}_{ij}, x^{**}_{ki} are non-negative integers (2-18)

$y_i = (0, 1)$ (2-18a)

where: y_i = 1 if the ith facility is built

= 0 otherwise

x^{*}_{ij} = flow of material from facility i to sink j

x^{**}_{ki} = flow of material from source k to intermediate point i

$c^{*}_{ij} = c_{ij} + r_j$ = unit cost associated with a transfer of material from facility i to sink j (dollars per unit)

c_{ij} = unit shipping cost from facility i to sink j (dollars per unit)

r_j = unit variable cost associated with using sink j (dollars per unit)

$c^{**}_{ki} = c'_{ki} + t_k + V_i$ = unit cost associated with transfer of material from source k to facility i (dollars per unit)

c'_{ki} = unit shipping cost from source k to facility i (dollars per unit)

t_k = unit variable cost associated with using source k (dollars per unit)

V_i = unit variable cost associated with using facility i (dollars per unit)

F_i = fixed charge for establishing facility i (dollars)

S_k = amount supplied at source k

$D_j{}^u$ = upper bound on amount demanded at sink j

$D_j{}^l$ = lower bound on amount demanded at sink j

Q_i = capacity of the i'th facility

m = number of proposed facility sites

n = number of demand areas

p = number of supply points

Inequality (2-15) expresses the requirement that flow from the source cannot exceed the supply of material there. Equation (2-16) states the conservation of flow requirement that the flow entering the i'th facility must be equal to the flow leaving it. In inequality (2-17) the fixed charge nature of facility location is expressed. If the i'th facility does not exist, $y_i = 0$ and no flow may take place through it. If the i'th facility does exist, $y_i = 1$ and flow up to Q_i may pass through it. Inequality (2-18) specifies that flow to sink j must be between the upper and lower bound on the capacity of the sink.

There are several additional constraints or side conditions that might be of interest in the above formulation. A budget constraint on the number or cost of the facilities to be built, or a mutual exclusivity constraint which allows a particular facility to exist only if another does not, might be included. As noted with sinks (disposal points) a lower bound requirement different from zero on the flow through a facility or a particular arc due to technical requirements might also be imposed. The side conditions will be discussed in detail later in this chapter.

C. Method of Solution

It is desired to find a solution to Problem One that will give values to the variables x^{*}_{ij}, x^{**}_{ki} and y_i so that the constraint set is satisfied and the objective function is minimized. One means for doing this would be to set up the problem as an integer linear programming problem and attempt to solve it by a cutting plane method such as that of Gomory (1958). However, for an M-facility, N-demand-area, P-supply-point problem there are

$$2(M + N + P) \text{ constraints}$$

and $$M \times N + M \times P + M \text{ variables which are}$$

required to be integer.

Thus for a problem of M=10, N=10, P=10, the integer program would have 60 constraints and 210 variables. Solution of the corresponding linear programming problem with the integer constraints relaxed will not necessarily yield integer solutions.

However, there are some linear programming problems which have special structure that may be related to a network, which may be solved optimally by procedures other than the simplex method of linear programming in a much shorter time and yield integer solutions. An example is the trans-shipment problem which represents a supply-demand relationship where demands for a product at some points are satisfied by shipments from sources of the product through intermediate points. The resulting linear programming problem may be shown to have the form of a network, and finding the minimum cost maximum flow through the network gives the solution to the trans-shipment problem.

Consider Problem One. It has some elements of network structure, but it may not be represented as a network problem. Inequalities (2-15), (2-16) and (2-18) exhibit an incidence matrix which by themselves would form a trans-shipment problem and would yield integer solutions. The addition of inequality (2-17) which has a coefficient of one variable that may be different from zero or one, destroys this property. However, a related but similar problem may be written in a form that will allow it to be solved by a network algorithm. Consider the problem of determining the flow through the system from supply source through intermediate facilities to demand areas if the facilities to be built are known. This implies that the values for the y_i's (the variable which determines which facilities are built) are known and given. Let S equal the set of indices representing the values of $y_i=1$. Then the problem may be written in the following form:

Problem One (a)

$$\text{Minimize:} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c^{*}_{ij} x^{*}_{ij} + \sum_{i=1}^{m} \sum_{k=1}^{p} c^{**}_{ki} x^{**}_{ki} + \sum_{i \in S} F_i \qquad (2\text{-}19)$$

subject to the constraints:

$$\sum_{i=1}^{m} x^{**}_{ki} \geqslant S_k \qquad k=1, 2, \ldots, p \qquad (2\text{-}20)$$

$$\sum_{k=1}^{p} x^{**}_{ki} \leqslant Q_i \qquad i \in S \qquad (2\text{-}21a)$$

$$\sum_{k=1}^{p} x^{**}_{ki} = 0 \qquad i \notin S \qquad (2\text{-}21b)$$

$$\sum_{j=1}^{n} x^{*}_{ij} = \sum_{k=1}^{p} x^{**}_{ki} \qquad i=1, 2, \ldots, m \qquad (2\text{-}22)$$

$$D_j^{u} \geq \sum_{i=1}^{m} x^{*}_{ij} \geq D_j^{l} \qquad j=1, 2, \ldots, n \qquad (2\text{-}23)$$

$$x^{*}_{ij}, x^{**}_{ji} \geq 0 \qquad (2\text{-}23a)$$

where the variables are defined in the same manner as for Problem One, and

S = the set of indices for which $y_i=1$.

This problem has the form of a network problem as described by Ford and Fulkerson (1962), and can be solved by finding the minimum cost maximum flow in the network. The method for finding the minimum cost maximum flow through the network is the out-of-kilter algorithm which was developed by Fulkerson (1961).[1] One of the computer codes for the IBM 7000 and 360 series computers is in Fortran IV by Clasen, and is available through the IBM Share Distribution System.[2]

It should be noted that without the upper and lower bound on demand at the sink expressed in inequality (2-23), the above problem would be a simpler trans-shipment problem rather than an out-of-kilter problem. However, the existence of the need for such constraints for this and other aspects of the problem means that an out-of-kilter problem rather than a simpler trans-shipment problem will be used for the basis for algorithmic development. The advantages of being able to formulate this subproblem as a network problem solvable by a network algorithm such as the out-of-kilter are twofold. First, network algorithms are very fast when compared to simplex based linear programming techniques. Second, the out-of-kilter algorithm may be started with any existing solution after constraints or costs have been changed. This restart capability will be of considerable interest in the solution scheme to be developed. Because of the network structure of the problem, both the network algorithm and a simplex algorithm will always yield integer solutions.

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[1] Those interested in following the development and proofs of the out-of-kilter algorithm are directed to Ford and Fulkerson (1962) or Fulkerson (1961). A good description of a more primary nature is found in Durbin and Kroenke (1967). A discussion of numerical techniques is found in Clasen (1968).

[2] IBM Share Distribution #3536, "Out-of-Kilter Network or Transportation Problem Solver", IBM, White Plains, New York.

The network structure of a location problem has been recognized by other authors. Efroymson and Ray (1966), and Spielberg (1969), while working on simpler location problems noted this structure and took advantage of it in a branch and bound scheme. In their case, the resulting structure was that of a transportation problem and the means of finding a lower bound on a branch might involve the solution of such a problem at each node of the branch. However, if the simplifying assumption is made that facilities do not have an upper bound on capacity, the transportation problem decomposes into a simple iterative scheme which is quite fast. Both authors attempted to save time by ignoring capacity constraints and using the simple iterative scheme. This limited their work to transportation problems.

The technique to be proposed below arose from the need to include both a trans-shipment network structure, and capacity constraints on facilities in the formulation of transfer station location problems. It was clear from the outset that this more general formulation would require greater computation time, but it was anticipated that the increased cost of finding a solution would be more than offset by the greater generality of the solution. Much of the success of the algorithm is based on the ability to restart the out-of-kilter algorithm at each stage, so that little computational effort was necessary to calculate successive lower bounds. Trans-shipment and transportation algorithms, with slight manipulation of the solution, also exhibit this restart capability.

D. A Sketch of the Algorithm

It is desired to solve Problem One, which is repeated here for convenience.

Problem One - The Capacitated Trans-shipment Facility Location Problem

$$\text{Minimize:} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c^{*}_{ij} x^{*}_{ij} + \sum_{i=1}^{m} \sum_{k=1}^{p} c^{**}_{ki} x^{**}_{ki} + \sum_{i=1}^{m} F_i y_i \qquad (2\text{-}24)$$

subject to the constraints:

$$\sum_{i=1}^{m} x^{**}_{ki} \geq S_k \qquad k=1, 2, \ldots, p \qquad (2\text{-}25)$$

$$\sum_{j=1}^{n} x^{*}_{ij} = \sum_{k=1}^{p} x^{**}_{ki} \qquad i=1, 2, \ldots, m \qquad (2\text{-}26)$$

$$\sum_{k=1}^{p} x^{**}_{ki} \leq Q_i y_i \qquad i=1, 2, \ldots, m \qquad (2\text{-}27)$$

$$D_j^{u} \geq \sum_{i=1}^{m} x^{*}_{ij} \geq D_j^{1} \qquad j=1, 2, \ldots, n \qquad (2\text{-}28)$$

x^{*}_{ij}, x^{**}_{ki} are non-negative integers

$y_i = 0, 1$

where the notation is as previously stated.

This problem has two elements that make it difficult to solve. First, it is a fixed charge problem with the fixed charge variable, y_i, appearing in inequality (2-27) with a coefficient Q_i which may be different from plus or minus one. Second, an integer solution is required. However, as has been previously noted, if the values of the fixed charge variables, y_i, are known, then the resulting problem of finding the values of the flow variables, the x_{ij}^{*}'s and the x_{ki}^{**}'s, becomes a form of trans-shipment problem whose solution may be found by using the out-of-kilter algorithm to find the minimum cost maximum flow of the corresponding network. In order to solve Problem One, some approximations will be made.

The flow of materials through the i'th facility is given by

$$\sum_{k=1}^{p} x_{ki}^{**}$$

and the cost function of the i'th facility is

$$\begin{array}{ll} F_i + V_i\left(\sum_{k=1}^{p} x_{ki}^{**}\right) & \text{if the i'th facility exists, i.e. if } y_i=1 \\ 0 & \text{if the i'th facility does not exist, i.e. if } y_i=0 \end{array} \qquad (2\text{-}29)$$

The slope of this cost function is shown in Figure 2-1, along with a linear approximation of the cost. The approximation is arrived at in the following manner. Convert the fixed charge, F_i, into a unit charge by dividing by the capacity of the facility Q_i. Then the approximate cost function is

$$\left(\frac{F_i}{Q_i} + V_i\right)\left(\sum_{k=1}^{p} x_{ki}^{**}\right) \qquad (2\text{-}30)$$

and it underestimates the true cost function given in (2-29) except at two points where they are equivalent. These are when there is

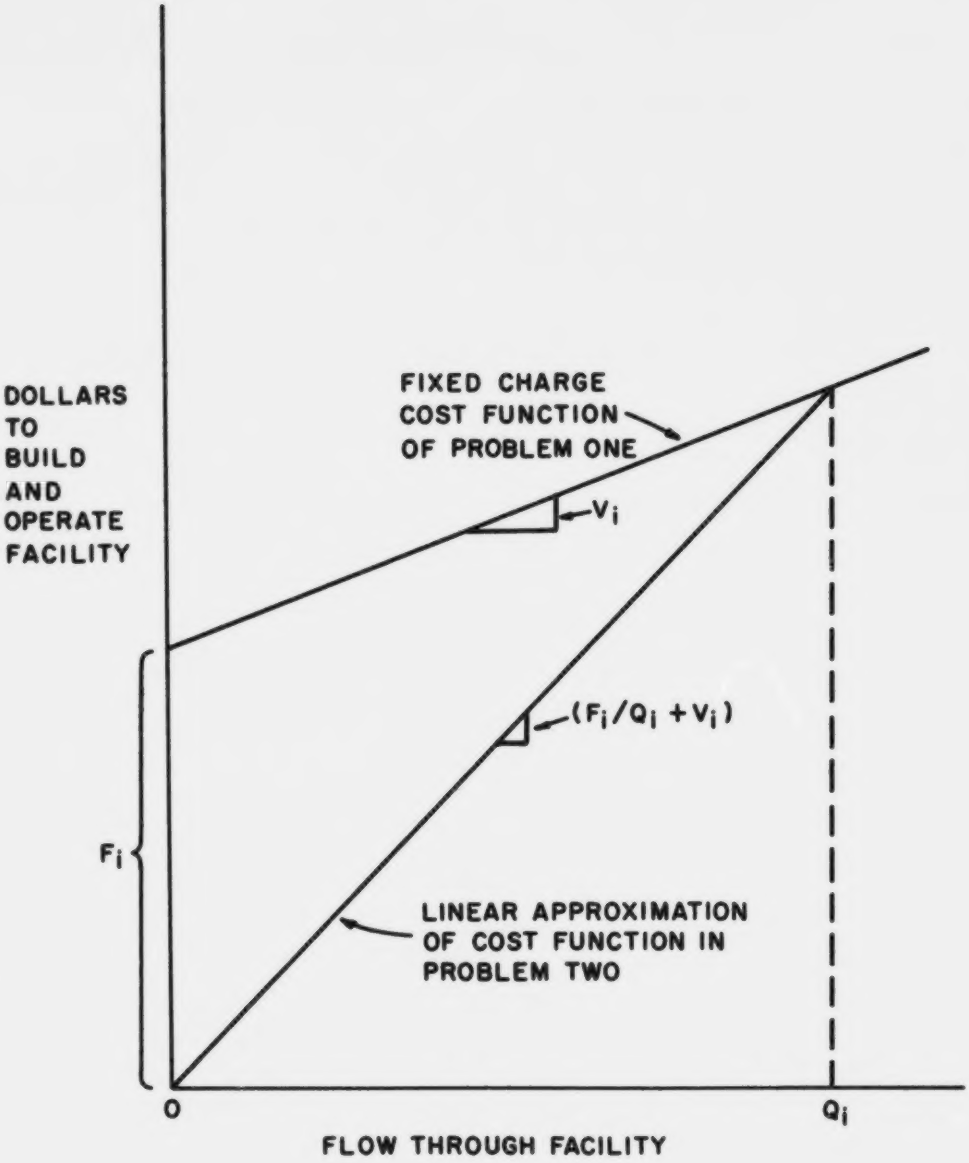


Figure 2-1. The Fixed Charge Cost Function of Problem One and Its Approximation in Problem Two.

no flow through the facility, and when the flow equals the capacity. The original idea for this approximation is due to Baumol and Wolfe (1958).

Suppose that all the facilities are given such an approximate cost function, and the constraint on the capacity of each facility (2-27) is adjusted to

$$\sum_{k=1}^{p} x_{ki}^{**} \leq Q_i \qquad i=1, 2, \ldots., m$$

as if all facilities existed. Then an approximation problem may be written as

Problem Two - Linear Approximation Problem

Minimize:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{*} x_{ij}^{*} + \sum_{i=1}^{m} \sum_{k=1}^{p} c_{ki}^{**} x_{ki}^{**} + \sum_{i=1}^{m} \frac{F_i}{Q_i} \left(\sum_{j=1}^{n} x_{ij}^{*} \right) \qquad (2\text{-}32)$$

subject to the constraints:

$$\sum_{i=1}^{m} x_{ki}^{**} \geq S_k \qquad k=1, 2, \ldots., p \qquad (2\text{-}33)$$

$$\sum_{j=1}^{n} x_{ij}^{*} = \sum_{k=1}^{p} x_{ki}^{**} \qquad i=1, 2, \ldots., m \qquad (2\text{-}34)$$

$$\sum_{k=1}^{p} x_{ki}^{**} \leq Q_i \qquad i=1, 2, \ldots., m \qquad (2\text{-}35)$$

$$D_j^{u} \geq \sum_{i=1}^{m} x_{ij}^{*} \geq D_j^{1} \qquad j=1, 2, \ldots., n \qquad (2\text{-}36)$$

$$x_{ij}^{*}, x_{ki}^{**} \quad \text{are positive integers}$$

$$y_i = (0, 1) \qquad (2\text{-}36a)$$

Problem Two has a linear objective function and the constraints take the form of a network problem solvable by the out-of-kilter algorithm.

Problem One and Problem Two have similar constraint sets so that a feasible solution to Problem Two is also a feasible solution to Problem One. However, because of the differences in the cost functions, the solution of Problem Two gives a lower bound for the cost of the optimal solution to Problem One. If the flow through each facility in Problem Two is

$$\sum_{k=1}^{p} x_{ki}^{**} = Q_i \text{ or } 0 \qquad i=1, 2, \ldots., m$$

then the cost of Problem Two is the same as that of Problem One and an optimal solution has been found. If for some facility the flow through the facility is

$$0 < \sum_{k=1}^{p} x_{ki}^{**} < Q_i$$

then a lower bound and not an exact solution has been found for Problem One and the search for the optimal solution must continue. This is accomplished by a method of search through the set of feasible solutions by either branch and bound or implicit enumeration methods. While both are search methods based on branching and the use of bounds for eliminating areas of search, the two methods do differ somewhat in the search procedure. Implicit enumeration implies that a branching scheme will be set up so that all possible feasible solutions will be enumerated either implicitly or explicitly. Bounds are used to avoid searching branches which are obviously dominated by existing feasible solutions. Examples of such techniques are Glover (1965), Balas (1965), Balas (1967) and Geoffrion (1966, 1967). Branch and bound implies that the set of feasible solutions is divided into mutually exclusive sets and lower bounds estimated until an optimal solution has been found and verified. In implicit enumeration, the goal is to enumerate all solutions while in branch and bound the goal is to subdivide into subsets until lower bound estimates show that an optimal solution has been found. Examples of branch and bound techniques are the work of Land and Doig (1960) and Little (1963). An example of implicit enumeration in the field of environmental engineering is a solution to a water pollution control problem by Liebman and Marks (1968).

The method of solution chosen to solve Problem One is branch and bound. The basic reasoning behind the technique is that it is possible to solve a highly structured problem by solving less highly structured surrogates. A feasible solution to the surrogate problem is always a feasible solution to the highly structured problem being approximated as well. However, the value of the objective function of the surrogate is not necessarily the value of the objective function of the problem it approximates with the same feasible solution. Because of the linear approximation used, the cost of the surrogate is lower than or equal to the cost of solving the original problem. Define an exact solution as an optimal, feasible solution to the surrogate where the value of the objective function for both the surrogate problem and the original problem are the same. A lower bound solution represents the case when the cost of the surrogate problem is lower than the cost of the original problem. The surrogate problems are generated in such a manner that the solution space for the original problem is subdivided into mutually exclusive collectively exhaustive sets. The solution of the problem representing each set is a lower bound for that set. The optimal solution is found when the smallest of the lower bounds for all the sets is an exact solution to the original problem.

Consider solving Problem One. The less constrained Problem Two will be solved as a surrogate for it. A solution is generated by the out-of-kilter algorithm and the cost and exactness of the solution to Problem One are investigated. If, for each facility the flow through it is exactly zero or the capacity of the facility, an exact solution to Problem One has been found as well. Since this solution represents the only lower bound estimate of Problem One, and it is feasible to Problem One, the optimal solution of the original problem has been found as well. The algorithm terminates. However, suppose that for some facility, i^*, the flow is greater than zero but less than capacity. Choose this facility to branch on.

In all feasible solutions to Problem One only two situations may occur regarding i^*. Either $y_i^*=1$ and the i^* facility exists, or $y_i^*=0$ and the i^* facility does not exist. Construct two new problems: Problem Two(a), (i^*), is Problem Two plus the constraint $y_i^*=1$. In order to set up this problem, the linear approximation associated with i^*, $\frac{F_i^*}{Q_i^*} + V_i^*$, is replaced by its true cost which is a constant, F_i^*, plus a variable cost of V_i^*. No change is made in the constraint matrix since it has already been assumed that all $y_i=1$ in the original formulation of Problem Two. Since F_i^* is a constant, it is simply added to the cost of the solution of Problem Two(a), (i^*), found by solving the resulting network using the out-of-kilter algorithm. Since Problem Two(a), (i^*), is more constrained than Problem Two, its cost must be greater than or equal to the cost of Problem Two. Note that the flow through the i^* facility may take on any value between zero and capacity. The correct cost will be assigned, since the fixed charge for the facility is included in the cost, and the variable charge is a function of flow through the facility. Now construct Problem Two(b), i^*, which is Problem Two plus the constraint $y_i^*=0$. This may be handled in two ways. Either set the variable cost at facility i^* to infinity or set the capacity at facility i^* to zero. Both procedures will produce the result of blocking flow through facility i^*. The solution to the two problems represents lower bounds of two mutually exclusive sets of feasible solutions to the original problem. The lower bound with the lowest objective function is examined for exactness to Problem One. If it is exact, it is optimal. If not, a new variable, i^{**}, is chosen for branching, and two new problems are set up. One problem has $y_i^{*}=1$, $y_i^{**}=1$, and the other has $y_i^{*}=1$ and $y_i^{**}=0$. The cost of these solutions, along with the remaining lower bound from the first branch, are searched for a minimum once more, and the exactness check is made. The procedure is continued until the optimal solution is found. The procedure must terminate since the process of adding constraints continues only until total enumeration of all possible feasible solutions has been found.

To present the subproblem solution in a general form, define:

S^* = the set of all indices representing facilities

S^1 = the set of facilities constrained to be built (i.e. $y_i=1$)

S^o = the set of facilities constrained not to be built (i.e. $y_i=0$)

$\underline{S}$ = the set of facilities not yet constrained

Then $S^* = S^1 \cup S^o \cup \underline{S}$

and $S^1 \cap S^o \cap \underline{S} = \phi$;

that is, the sets are mutually exclusive and totally exhaustive.

Then a general subproblem may be written as:

Minimize:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} c^*_{ij} x^*_{ij} + \sum_{k=1}^{p} \sum_{i=1}^{m} c^{**}_{ki} x^{**}_{ki} + \sum_{i \in \underline{S}} \frac{F_i}{Q_i} \left(\sum_{k=1}^{p} x^{**}_{ki} \right) +$$

$$\sum_{i \in S^o} \infty \times \left(\sum_{j=1}^{n} x^*_{ij} \right) + \sum_{i \in S^1} F_i \qquad (2\text{-}37)$$

subject to the constraints:

$$\sum_{i=1}^{m} x_{ki}^{**} \geq S_k \qquad k=1, 2, \ldots, p \tag{2-38}$$

$$\sum_{k=1}^{p} x_{ki}^{**} \leq Q_i \qquad i \in \underline{S} \cup S^1 \tag{2-39}$$

$$\sum_{j=1}^{n} x_{ij}^{*} = \sum_{k=1}^{p} x_{ki}^{**} \qquad i \in S^* \tag{2-40}$$

$$D_j^u \geq \sum_{i=1}^{m} x_{ij}^{*} \geq D_j^1 \qquad j=1, 2, \ldots, n \tag{2-41}$$

$$x_{ij}^{*},\ x_{ki}^{**} \text{ are non-negative integers} \tag{2-41a}$$

and all symbols are as previously defined.

The ease of keeping track of the various solutions in a computer is aided by the use of associative lists which allow solutions to be kept in order of increasing cost, and makes location of the lowest cost solution relatively simple.

The choice of a variable on which to branch can make a great deal of difference in the speed of finding the solution. The rule used in the procedure for solving Problem One involves searching the facilities which have flow greater than zero but less than the capacity of the facility. Denote the fraction of flow through facility i as f_i (defined as the flow divided by the capacity). Select for branching that facility whose value of f_i is closest to 0.5, which is the rule that Land and Doig (1960) suggest.

E. A Detailed Algorithm for Solving the Capacitated Trans-Shipment Facilities Location Model

1. The following data are given:

m = the number of proposed facility sites

n = the number of demand areas

p = the number of sources

For each facility site, i, the following are known:

F_i = the fixed cost of establishing the i'th facility

V_i = the unit variable cost of operating the established facility

Q_i = the capacity of the facility

For each demand area, j, the following are known:

D_j^u = the upper bound on quantity demanded at the j'th demand area

D_j^l = the lower bound on quantity demanded at the j'th demand area

r_j = unit variable cost of satisfying demand at j

For each supply point, k, the following are known:

S_k^u = the upper bound on quantity supplied at point k

S_k^l = the lower bound on quantity supplied at point k

t_j = unit variable cost of providing supply from point k

In addition, the $m \times n$ matrix c_{ij}, and the $p \times m$ matrix c^*_{ki} are assumed known where

c_{ij} = the unit cost of transportation from facility i to demand area j

c^*_{ki} = the unit cost of transportation from source k to facility i

2. **Initialization:**

Define S = the set of all indices of facilities that have been fixed at zero or one

$\underline{S}$ = the set of all indices not yet set

S^* = the set of all indices = $(1, 2, 3, \ldots, m)$

Then $S \cup \underline{S} = S^*$ $\qquad S \cap \underline{S} = \phi$ (the empty set)

Initialize $S = \phi$

$\underline{S} = S^* = (1, 2, 3, \ldots, m)$ = all indices representing facilities

3. Construct a graph in the following manner: There will be three sets of nodes (I, J, K) which make up the graph, plus a source node S#. The graph will be in circulation form[3] so that the source = sink = S#.

Define I = (set of nodes which represent the facility site alternatives)

For each of the m sites, a capacitated node is established. This is accomplished by representing the site as two nodes with a directed arc joining them (see Figure 2-2). The upper bound capacity of the arc is the capacity, Q_i, of the facility it represents. The lower bound capacity is zero. The unit cost on the arc is Z_i, which will be defined later.

Define J = (set of nodes which represent the demand areas)

Define K = (set of nodes which represent the supply points)

---

[3] Circulation form means an arc exists from the sink back to the source.

Arcs connecting the nodes:

From S# to set K: Define a directed arc from S# to each node in K. The unit cost on the arc is t_k. The upper bound on capacity of the arc is S_k^u and the lower bound is S_k^l. Assume the upper bound is equal to the lower bound.

From set K to set I: Define a directed arc from each node in K to the receiving end of each capacitated node in I. The cost of the arc is c_{ki}^* and the upper and lower bound capacity of the arc may be set as desired. Assume the upper bound is infinity and the lower bound is zero.

From set I to set J: Define a directed arc from the transmitting end of each capacitated node in I to each node in J. The unit cost of the arc is c_{ij}, and the upper and lower bound capacity may be set as desired. Assume the upper bound is infinity and the lower bound is zero.

From set J to S#: Define a directed arc from each node in J to S#. The unit cost on the arc is r_j, and the upper bound and lower bound capacities are respectively D_j^u and $D_j l$.

Figure 2-3 shows a graph representation for m=3, n=3, k=2.

4. Establish the unit cost, Z_i for the capacitated nodes in set I.

Define y_i = a decision variable

Define y_i = 1 if the i'th variable is in the set S and if
the i'th facility is to be built

Define $y_i = 0$ if the i'th variable is in the set S and
the i'th facility is not to be built, or
if the i'th variable is in the set $\underline{S}$

Define $Z_i = 0$ if $y_i=1$ and $i \in S$

Define $Z_i = \infty$ if $y_i=0$ and $i \in S$

Define $Z_i = \frac{F_i}{Q_i}$ if $y_i=0$ and $i \in \underline{S}$

5. Find the minimum cost maximum flow through the graph using the out-of-kilter algorithm. Define this cost as CGRAPH. If this is the first time through this step and the attempt to solve the graph using the out-of-kilter method indicates no feasible solution, no feasible solution to the problem exists. TERMINATE. If not, continue.

6. Define the total cost of the solution, COST, as

$$\text{COST} = \text{CGRAPH} + \sum_{i \in S} F_i y_i$$

7. Examine the solution to see if it is exact:

   Define X_i as the flow through the capacitated node
   representing the i'th facility

   The solution is exact if for all $i \in \underline{S}$

   $$X_i = 0 \quad \text{or} \quad X_i = Q_i$$

   If the solution is not exact, find the index of one of the members of the set $\underline{S}$ for which $0 < X_i < Q_i$, as the next candidate for branching. Select that member for which $\frac{X_i}{Q_i}$ is nearest to 0.5.

8. Store information about the solution. File on a list ranked on the cost of the solution information about the exactness, the members of the S set and the values to which they are set, and the next candidate for branching.

9. Remove the first problem on the list, i.e. the lowest cost solution found so far. If the solution is exact, it is optimal. TERMINATE.

10. If the solution is inexact, set up subproblems (a) and (b):

    (10a). For subproblem (a):

    Set $y_i=1$ for the index stored as the candidate for branching and switch that index from $\underline{S}$ to S

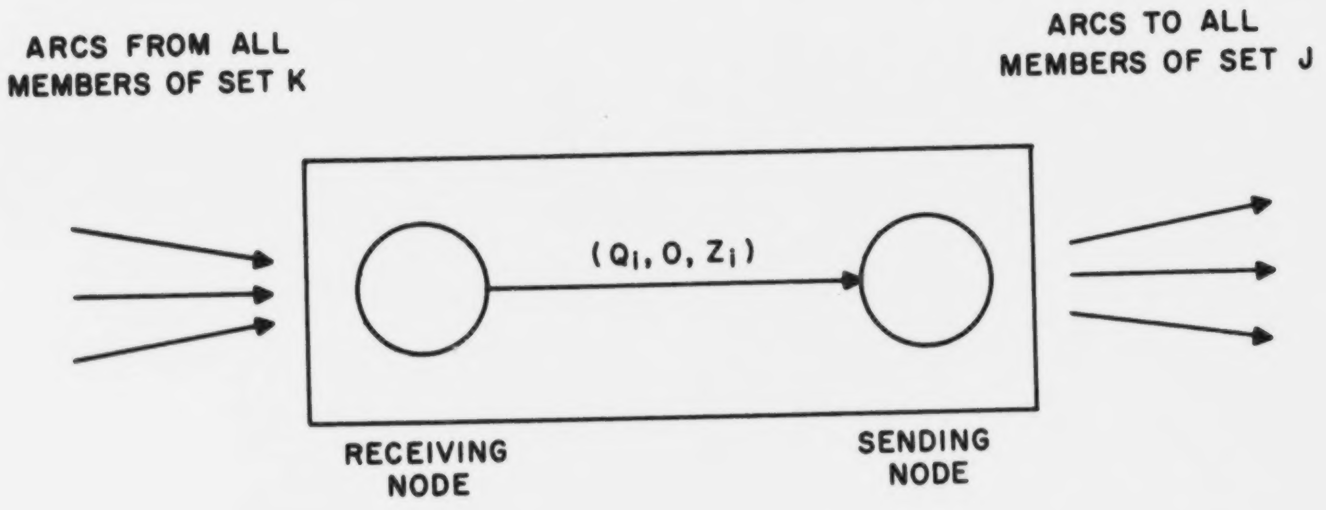
    Then do steps 4 through 8 and return to do subproblem (b) in (10b)

    (10b). For subproblem (b):

    Set $y_i=0$ for the same index as in (a) and switch that index from $\underline{S}$ to S

    Then do steps 4 through 8.

    Go to step 9.


ARCS FROM ALL
MEMBERS OF SET K
ARCS TO ALL
MEMBERS OF SET J
(Q_i, O, Z_i)
RECEIVING
NODE
SENDING
NODE

Q_i = UPPER BOUND ON FLOW THROUGH FACILITY i.

Z_i = UNIT COST ASSOCIATED WITH USING FACILITY i.

Figure 2-2. A Capacitated Node Representing Flow Through Facility i.

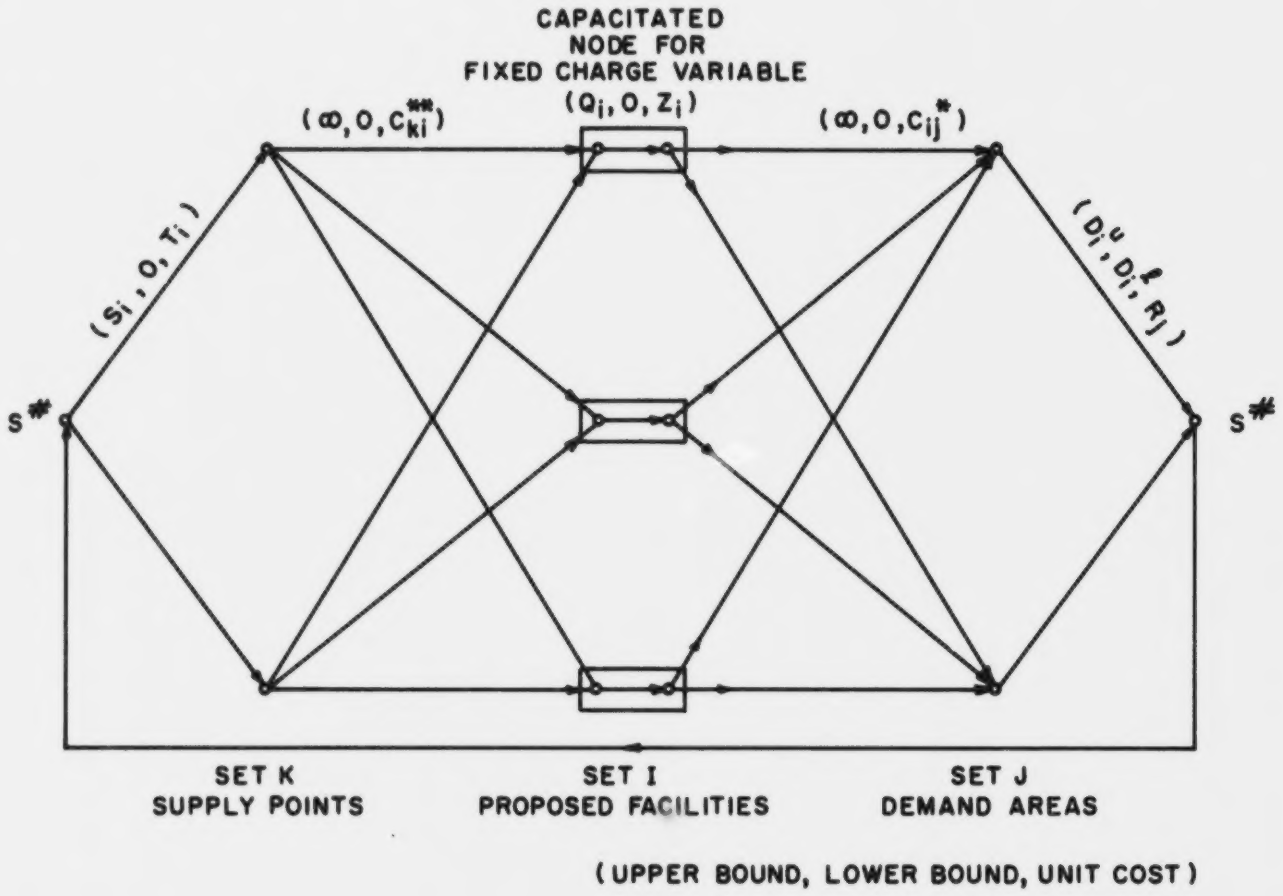

CAPACITATED
NODE FOR
FIXED CHARGE VARIABLE
(Qi, O, Zi)
(∞, O, Cki**)
(∞, O, Cij*)
(Si, O, Ti)
(Dju, Djℓ, Rj)
S#
S#
SET K
SUPPLY POINTS
SET I
PROPOSED FACILITIES
SET J
DEMAND AREAS
(UPPER BOUND, LOWER BOUND, UNIT COST)

Figure 2-3. A Graph Representation for a Problem with m=2, n=3, k=2.

F. Computational Results Using the Algorithm

The algorithm as described was coded in ASA Fortran IV and executed on an IBM 7094 computer. The out-of-kilter algorithm used to solve the lower bound problem at each branch was a Share Library Distribution Program written by Clasen; the list processing subroutine package used in keeping track of the various solutions was written by Bellmore (1966). In Chapter V results based on the use of data from example problems in solid waste collection are presented. General impressions of this type of algorithm gained by running it with randomly generated data are as follows: The usual case for branch and bound algorithms is that they are data dependent. The same size problem with different data can lead to very different running times. In the case of the algorithm tested, the problems that ran the fastest for a given size were those where the facility costs were sufficiently high so that only a few facilities were built. From computational experience with such algorithms, it has been found that problems where the optimal solutions are to build either a few or all of the facilities, generally run the fastest. Problems where there is a delicate trade-off between facility and transportation costs, so that several facilities are built, tend to take longer to derive optimal solutions. Further, the number of out-of-kilter problems solved in obtaining the optimal answer definitely exhibited the value of the algorithm's restart capability. As the number of out-of-kilter solutions increased, the time per solution decreased.

A possible limiting factor on the use of this algorithm is that the amount of storage space needed within the computer for a problem is quite large.

Let F = the number of proposed facilities
D = the number of demand areas
S = the number of supply points

Then the number of arcs and nodes in the resulting graph is:

$$\text{Number of arcs} = D(1 + F(1 + S))$$

$$\text{Number of nodes} = D + 2F + S$$

The maximum size problem that could be run on the 7094 with its 32,768 word memory was 300 nodes and 1500 arcs. The number of feasible alternatives is dependent on the number of facilities being investigated. However, comparing two solutions with the same number of facilities but different numbers of supply points will show that there is a significant difference in time to solve the out-of-kilter algorithm due to its larger number of arcs. The computational results are shown in Table 2-1.

Table 2-1.

Results of Trial Runs of the Algorithm with Randomly Generated Data

F	D	S	No. of Alternatives	No. of Arcs	No. of Nodes	No. of OKA's**	7094 Time (minutes)
6	12	2	64	104	27	8	0.14
12	10	2	4,096	194	35	8	0.20
* 10	30	3	1,024	382	73	8	0.25
* 10	30	2	1,024	382	53	8	0.54
15	20	3	32,000	388	54	3	0.28
30	20	3	10^{10}	733	84	15	1.01
10	20	3	1,024	293	44	8	0.22
20	20	3	10^{6}	503	64	6	0.37
30	25	2	10^{10}	866	88	30	1.24
+ 35	10	3	10^{12}	478	84	30	1.60
+ 35	10	3	10^{12}	478	84	11	0.84
+ 35	10	3	10^{12}	478	84	6	0.56
50	10	3	4×10^{15}	713	114	20	1.19

* These two solutions have decreasing fixed costs (in order given).

+ These three solutions have increasing fixed costs (in order given).

** Number of times out-of-kilter algorithm applied.

G. Augmenting the Algorithm to Consider Additional Side Conditions

There are several additional constraints that might also be added to the problem to give more generality. Consider the following conditions and the way the algorithm must be adjusted to allow for their inclusion.

1. Mutual exclusivity constraints of the form of

$$\sum_{i \in E} y_i \leq 1 \qquad (2\text{-}42)$$

where E is some subset of facilities for which only one member of the subset may be constructed. This constraint is used in the case where the set E represents several treatment alternatives that might be built at a possible site. Then either none or one of the possible alternatives may be built. It might also be used as an informal budget or political constraint. If one facility is built at one site, another cannot be built at a different site. The way that this is handled in the algorithm is as an additional feasibility requirement. In order to be an exact feasible solution to Problem One, the solution to Problem Two must not only satisfy flow through facility constraints but the mutual exclusivity constraints as well. If it is violated, an exact solution has not been found. A fast and convenient way to set up a branching rule for this procedure is the following: Suppose that the variable chosen for branching is i^* and $i^* \in E$. Since no other member of E may appear in a feasible solution if $y_i{}^*$ is set to one, set $y_i{}^* = 1$ and $y_i = 0$ for all $i \in E$ but not equal to i^*. Switch all indices $i \in E$ from $\underline{S}$ to S.

2. Budget constraints of the form

$$\sum_{i \in W} a_i y_i \leq B \tag{2-43}$$

where W may be a subset of the facilities or the entire set. The mutual exclusivity constraint in (2-42) is a special case of (2-43) (with all $a_i = 1$, and $B = 1$). Each time a member of the set E is a candidate for branching, a check is made to see if establishing that facility will violate the budget constraint. If it will not, then branching may proceed. If it will, the variable is only allowed in the solution at a value $y_i = 0$, and the search for another variable on which to branch is carried out.

3. Upper and lower bounds on flow through a facility if it exists. Equipment may exist for which a certain minimum use is necessary to keep it from deteriorating. The constraint takes the form

$$Q_i^{\,l}\, y_i \leq \sum_{k=1}^{p} x_{ki}^{**} \leq Q_i^{\,u}\, y_i \tag{2-44}$$

where $Q_i^{\,l}$ and $Q_i^{\,u}$ are respectively the lower and upper bound on the capacity of the facility if it exists. The change in the algorithm that will allow this condition is in step 4. When Z_i, the cost of the arc, is defined, the capacity of the arc is defined as well.

Substitute these rules in step 4.

$Z_i = V_i$ and the upper and lower bounds are Q_i^u and Q_i^l if $i \in S$ and $y_i = 1$

$Z_i = \infty$ and the upper and lower bounds are zero if $i \in S$ and $y_i = 0$

$Z_i = \frac{F_i}{Q_i} + V_i$ and the upper and lower bounds are Q_i^u and zero if $i \in \underline{S}$

4. Upper and lower bounds on individual arcs such as

$$G_{ij}^{l} \leq x_{ij}^{*} \leq G_{ij}^{u} \qquad (2\text{-}45)$$

may be added without any change in the algorithm since the out-of-kilter algorithm allows such restrictions on arcs.

CHAPTER III. LARGE SCALE SINGLE AND MULTICOMMODITY ROUTING IN GIVEN NETWORKS

Introduction

In this chapter, the problem of routing a flow of commodities from sources through intermediate points to sinks in given networks is approached. As with the facility location problem of Chapter II, the assumption is that the capacity of the vehicle which comprises the means of transport is smaller than the quantity of material to be moved, so that the small scale collection problems of routing between sources for collection, or between sinks for distribution, may be avoided. In the narrowest sense, finding the flow through a given network is a simplification of the facility location problem and may be readily solved by the use of a transportation or transshipment algorithm. However, there are often additional considerations that might be introduced to the problem which increase the generality of the formulation and the difficulty of finding solutions. The first such consideration is the addition of time and capacity constraints on vehicles, budget constraints on certain types of expenditures and constraints on manpower allocation. Work in the general area has been done by Garvin et al. (1957) and Barton (1967), and work in the specific area of solid waste collection has been done by Wolfe and Zinn (1967) and Anderson (1968). Another area of interest is in the use of a common vehicle fleet to carry several distinct commodities between supply and demand points for the individual commodities. The work of Szwarc (1967) on multi-commodity transportation problems has been extended in this dissertation and will be discussed. Another area of interest is a problem considered by Ford (1958), Bomberault and White (1968) and White (1968) called the dynamic trans-shipment problem, in which it is desired to route vehicles not only in a spatial sense but in a temporal sense as well. This work finds application in the scheduling of empty railroad cars and may have application in solid waste collection.

A. Literature Review

The earliest report in the literature of a vehicle routing problem through a given network with added conditions was made by Garvin et al. (1957) in an article showing applications of linear programming in the oil industry. Included in the problems surveyed was that of routing vehicles from bulk refineries to service stations. The problem statement is as follows:

There is a given set of stations, k, each with a demand for a product D_k, and located at known distances from the bulk refinery and from all other stations. There are several different types of trucks available for delivering to the stations, each with a capacity C_s, a given unit cost of operation and a given number of hours per time period when that type of truck may be used. The problem is to find the delivery schedule that will meet the demands for the product at the stations within the capacity and time constraints while minimizing the transportation costs.

For this particular problem, it is assumed that any of the demands are greater than any of the vehicle capacities. The authors presented a linear programming formulation as follows:

$$\text{Minimize:} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{s=1}^{p} c_{ijs} x_{ijs} \tag{3-1}$$

$$\text{subject to:} \quad \sum_{i=1}^{n} y_{ijk} = \sum_{u=1}^{n} y_{juk} \qquad \begin{array}{r} k=1, 2, \ldots, n \\ j=1, 2, \ldots, n \\ \text{but } k \neq j \end{array} \tag{3-2}$$

$$\sum_{i=1}^{n} y_{ikk} = D_k \qquad k=1, 2, \ldots, n \qquad (3\text{-}3)$$

$$\sum_{k=1}^{n} \sum_{j=1}^{n} y_{ojk} = \sum_{k=1}^{n} D_k \qquad (3\text{-}4)$$

$$\sum_{i=1}^{n} x_{ijs} = \sum_{u=1}^{n} x_{jus} \qquad \begin{array}{l} j=1, 2, \ldots, n \\ s=1, 2, \ldots, p \end{array} \qquad (3\text{-}5)$$

$$\sum_{k=1}^{n} y_{ijk} \leq \sum_{s=1}^{p} C_s x_{ijs} \qquad \begin{array}{l} i=1, 2, \ldots, n \\ j=1, 2, \ldots, n \\ \quad \text{but } k \neq j \end{array} \qquad (3\text{-}6)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} h_{ijs} x_{ijs} \leq h_s \qquad s=1, 2, \ldots, p \qquad (3\text{-}7)$$

$$x_{ijs},\ y_{ijk} \quad \text{are non-negative integers} \qquad (3\text{-}7a)$$

where:

c_{ijs} = cost per unit for shipping from i to j in vehicle type s

x_{ijs} = the number of vehicles of type s sent from i to j

y_{ijk} = the quantity shipped from i to j with final destination k

D_k = quantity demanded at k

C_s = carrying capacity of s'th type vehicle

h_{ijs} = time required to go from i to j in vehicle type s

h_s = time that an s type vehicle can be used

n = number of customers (or cities)

p = number of vehicle types

Expression (3-1) specifies the minimization of the cost of vehicle operation. Equations (3-2), (3-3) and (3-4) are a set of trans-shipment constraints and exhibit an incidence matrix. Equation (3-2) specifies that all material shipped into a node destined for sink k must equal the amount of material shipped out of the node destined for sink k. Equation (3-3) is a destination constraint which sets the demand at the k'th station, and Equation (3-4) is a source constraint specifying the amount to be shipped. Equation (3-5) expresses a node constraint on trucks by specifying that the number of trucks of type s that enter a station must equal the number that leave it. Inequality (3-6) relates material trans-shipment to the capacity available to ship it, while inequality (3-7) specifies a time constraint on the availability of different types of vehicles.

The authors have ignored the integer constraints (3-7a) and solved the problem by using a linear programming code with rounding of variables that take non-integer values. The problem very quickly becomes unwieldy for any reasonable number of sources and sinks. For n sinks and s different types of service vehicles there are

$$2n^2 + ns \text{ constraints}$$

and

$$n^3 + n^2 s \text{ variables.}$$

A 10 sink problem has more than 200 constraints and 1100 variables. The authors did note the network structure of part of the constraint set and suggested that some improvement in running time might be gained by exploiting it. So far, however, nothing further has appeared in the literature. The linear programming approach continues to appear as bigger, faster codes become available. Barton (1967), in work on aircraft scheduling and routing, wrote essentially the same formulation and used the same technique to solve it. In this case, C.E.I.R.'s LP/90 code and an IBM 7094 allowed him to solve a 202 row, 515 column problem in 16 minutes. The solution is reported to have non-integer elements which are rounded to integer.

In application of models of this form to problems of solid waste collection, the sources are the locations from which solid waste is to be collected, the sinks are disposal points and the intermediate points are transfer stations where some form of processing such as incineration or compacting might take place in addition to the transfer operation. Arcs joining these points represent alternative routes or methods by which the material may pass from sources to sinks. Solving the problem will show the overall cost of collection and which alternative routes are used to obtain it. Some authors have looked at the problem of modeling the overall collection system in this manner. Wolfe and Zinn (1967) resorted to linear programming to solve the problem for a small example problem. The model they proposed would be an ordinary transshipment model which might then be solved by a primal-dual algorithm quickly, except for problems that arise in the representation of incinerators. Any waste flow that enters this node is decomposed into several different forms whose amounts are given proportions of the inflow into the node. This means that flows leaving the node have coefficients between zero and one and an incidence matrix no longer exists. Since there are advantages to using a primal-dual algorithm over a simplex algorithm, Anderson (1968) proposed an out-of-kilter algorithm for solving this problem which allows flow from nodes to be separated into given proportions. He showed a few examples solved by hand and did not indicate if the algorithm had been coded. Thus, while it would be expected that such an algorithm would be much faster than solving the same problem by a simplex linear programming code, there is no evidence yet to support it.

In many cases in vehicle routing, it is desired to consider the use of a common vehicle fleet to move more than one commodity between supply and demand points for a commodity. This is particularly true in solid waste collection, since many different classes of wastes such as residential, industrial, construction, salvageable materials, ashes, leaves and even Christmas trees are presently collected separately. Adding multicommodity aspects to such problem formulations as Garvin et al. (1957) proposed for single commodity flow may be done quite simply. Additional constraints defining the shipment network requirements for the additional commodities and how the commodities interact in their competition for shipment vehicles are easily formulated and added to the problem. However, since the formulation is already so complex that even a small problem is unmanageable in terms of computation technique and time, the addition of more commodities makes the formulation unworkable.

To be able to deal efficiently with a multicommodity routing problem it will be necessary to drop all consideration of additional constraints such as those on time, vehicles and work rules which obscure the simple network structure of the problem. This leads to the formulation of multicommodity transportation or trans-shipment problems and to solution methods which will prove faster than linear programming using the simplex method. Szwarc (1967) formulated and presented a solution technique for a multicommodity transportation problem which he calls the truck assignment problem. He formulated it in the following manner:

There exist m production sites each capable of producing p commodities, and n demand locations where there is a demand for each of the commodities. The demand for a commodity is such that the entire capacity of a vehicle is allocated to only one commodity and is destined for one demand location only. Thus the problem of routing a vehicle between supply locations or demand locations is avoided and a vehicle's assignment will consist of loading a commodity at a production source, delivering its contents to one demand location and returning empty to a production source for reassignment. The objective is to find the vehicle assignments that will minimize the total distance traveled while meeting the supply-demand constraints.

This problem, because of the requirements that a vehicle traveling from source to sink may carry only one commodity and that the return from the sink to the source is done empty, is a far easier, less complicated problem than the general multicommodity flow problems considered by Ford and Fulkerson (1958). Each arc in the traditional multicommodity flow problem may carry many different commodities at the same time. But with the special formulation, each arc from a source of a commodity to a sink of a commodity may carry only that commodity. Thus the problem may be decomposed into a series of simpler single commodity problems in which loaded vehicles are shipped from sources to sinks, and empty vehicles from sinks back to sources. Arcs representing flow between sources and sinks of different commodities are not allowed. However, arcs from the sinks back to the sources represent empty trucks and any source may be reached from any sink.

Szwarc presents no mathematical formulation of the problem, but proceeds to a solution method based on solving two separate transportation problems for an enlarged network in which a source of several commodities becomes a distinct source for each of the commodities. Sinks are treated in the same manner. If there are m production sources each supplying p commodities and n demand sites, then he solves first an mp by np transportation problem based on routing full trucks from sources to demand sites, with routes between supply and demand for different commodities blocked out. Then, the problem of routing the empty vehicles back to the source is presented as another transportation problem. A procedure is then shown which reconciles the two solutions into a set of vehicle assignments. A proof of optimality is included in the paper. This is a very quick, efficient method since solution time for even large transportation problems is quite small. A later section of this chapter gives an extension to multicommodity trans-shipment problems of the type described by Szwarc with the addition of many considerations which make the formulation more general.

Dynamic trans-shipment networks deal with the problem of the movement of goods and vehicles from location to location over time. Dealing with networks where there are time as well as spatial aspects to the problem often changes the type of analysis performed. The concern may shift from routing to finding the maximum flow through a point that may occur in a given time sequence. The routing of vehicles must specify not only which arcs of the network are traveled, but in what time period as well.

The first work to appear in the literature on dynamic flow problems was by Ford (1958) and is also reported in the textbook by Ford and Fulkerson (1962). The problem was to find the maximum dynamic flow through a given network where the dynamic flow at a point is defined as the total flow that passes that point during a specified time period. The normal concept of a capacity of an arc in a static or time-independent problem becomes a capacity per unit time in the dynamic problem and the cost on the arc represents the travel time through it. Ford showed that there is a direct relationship between static and dynamic problems and that the maximum flow through the network as obtained by a trans-shipment algorithm could be used directly to construct the solution to the dynamic problem. Bomberault and White (1968) were interested in the problem of routing empty shipping containers from sources where they exist to demand for them. In this case demand exists in only specified time periods and there is both a travel time and a cost associated with travel along an arc. The authors wished to apply a travel cost to transfer between nodes as well as a time constraint and then minimize the cost of shipping the containers to meet the temporal demand for them. By enumerating the feasible routes that exist for a container to reach a demand point before or as it is needed, they construct an expanded network through which flows which minimize transportation costs may be found by the use of a trans-shipment algorithm, much in the spirit of the Ford and Fulkerson algorithm. White (1969) suggested an improved algorithm for solving the empty container problem, which involves decomposing into a nested sequence of subproblems and uses an out-of-kilter algorithm for solution.

Applications of the work on dynamic trans-shipment networks to problems of solid waste collection are of interest particularly in the scheduling of large scale vehicles used for transport from transfer sites to disposal areas. Such vehicles may be either trucks or railroad cars, and questions such as how many of what type are needed may be approached using this formulation.

B. Formulation and Solution of a General Multicommodity Truck Assignment Problem

Szwarc (1967) described a method for solving multicommodity transportation problems which is discussed in the preceding section. His work is limited in application because it cannot deal with transshipment networks. In the paper on his method, Szwarc did not give a mathematical formulation for the problem he was attempting to solve. However, such a formulation gives a hint as to how it may be extended to a far more general form, and solved optimally using an out-of-kilter algorithm.

Consider the following definitions:

Let

x_{ijk} = the number of vehicle loads of commodity k to be sent from production source i to demand site j

x^*_{ji} = the number of empty vehicles sent from demand site j back to production source i

c_{ijk} = cost per vehicle load of shipping commodity k from i to j

c^*_{ji} = cost per vehicle of returning an empty vehicle from demand site j to production source i

D_{jk} = demand in vehicle loads for commodity k at demand site j

S_{ik} = supply in vehicle loads for commodity k at production source i

m = number of production sources

n = number of demand sites

p = number of commodities

Then the general mathematical formulation for Szwarc's delivery problem is:

Minimize: $$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} c_{ijk}x_{ijk} + \sum_{j=1}^{m} \sum_{i=1}^{n} c^{*}_{ji}x^{*}_{ji} \qquad (3\text{-}8)$$

subject to: $$\sum_{j=1}^{n} x_{ijk} \leqslant S_{ik} \qquad \begin{array}{l} i=1,\ 2,\ \ldots.,\ m \\ k=1,\ 2,\ \ldots.,\ p \end{array} \qquad (3\text{-}9)$$

$$\sum_{i=1}^{m} x_{ijk} \geqslant D_{jk} \qquad \begin{array}{l} j=1,\ 2,\ \ldots.,\ n \\ k=1,\ 2,\ \ldots.,\ p \end{array} \qquad (3\text{-}10)$$

$$\sum_{k=1}^{p} \sum_{i=1}^{m} x_{ijk} = \sum_{i=1}^{m} x^{*}_{ji} \qquad i=1,\ 2,\ \ldots.,\ m \qquad (3\text{-}11)$$

$$\sum_{k=1}^{p} \sum_{j=1}^{n} x_{ijk} = \sum_{j=1}^{n} x^{*}_{ji} \qquad i=1,\ 2,\ \ldots.,\ m \qquad (3\text{-}12)$$

$$x_{ijk},\ x^{*}_{ji} \quad \text{are non-negative integers} \qquad (3\text{-}12a)$$

The objective function in (3-8) is to minimize the cost of loaded trips from source to site and of empty return trips. Inequality (3-9) requires that the amount of commodity k shipped from source i to all sites must not exceed the supply of commodity k at that source. Similarly, inequality (3-10) specifies that the total amount of commodity k shipped to j from all sources must be greater than or equal to the demand for commodity k at that site. Equations (3-11) and (3-12) are flow equations requiring respectively that the number of empty vehicles sent out of a demand site must equal the number of loaded vehicles arriving there, and that the number of loaded vehicles sent out of a source must equal the number of empty vehicles arriving there. Inherent in this formulation is that vehicles must return to a source after leaving a demand site. The integer requirement (3-12a) will be shown to be automatically satisfied since the problem may be set up in network form and the solution found by finding the minimum cost-maximum flow using the out-of-kilter algorithm. Figure 3-1 shows the graph corresponding with this problem for the case of m=2 sources, n=2 sinks, p=2 commodities.

The computational difference between the two methods is shown by the fact that the Szwarc method requires the solution of two 4x4 transportation problems and some manipulation with the solution. For a solution by the out-of-kilter algorithm a graph of np + mp + mpn + mn arcs and (m+n)(p+1) nodes (or, for the example case, 20 arcs and 12 nodes), must be solved, which will take longer to compute. There are several advantages of the out-of-kilter approach which make it a recommended procedure for general cases, however.

1. It is possible to consider any sort of routing network with intermediate facilities. In this case, it may easily be shown that adding intermediate node equations to the model formulation still leaves a graph solvable by the out-of-kilter algorithm, but not by Szwarc's algorithm. Since most vehicle routing systems, including solid waste systems, involve the consideration of intermediate points such an extension is of considerable analytic value.

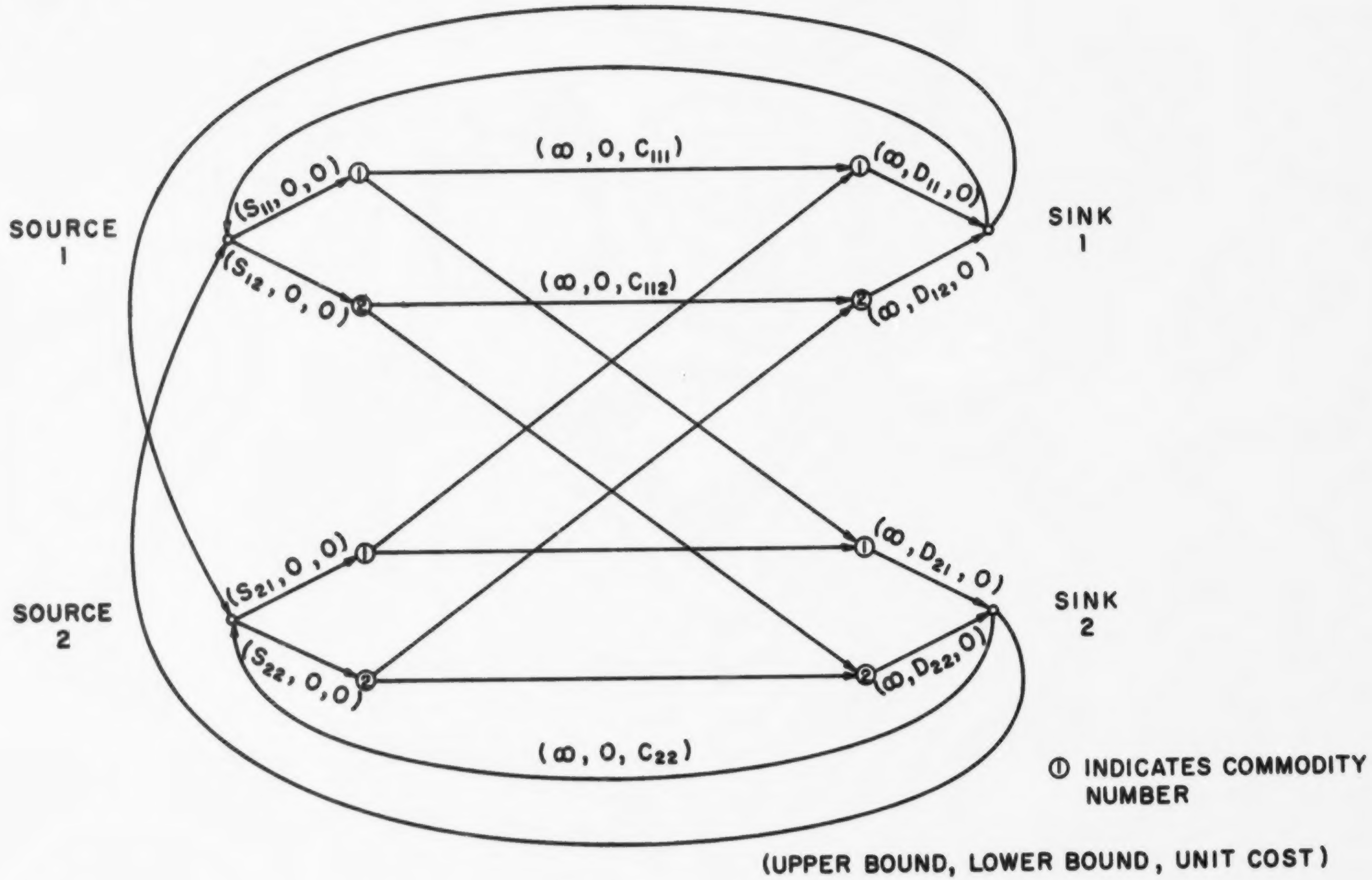

SOURCE 1
SOURCE 2
SINK 1
SINK 2
(S11, O, O)
(S12, O, O)
(S21, O, O)
(S22, O, O)
(∞, O, C111)
(∞, O, C112)
(∞, O, C22)
(∞, D11, O)
(∞, D12, O)
(∞, D21, O)
(∞, D22, O)
1
2
1
2
1
2
1
2
① INDICATES COMMODITY NUMBER
(UPPER BOUND, LOWER BOUND, UNIT COST)

Figure 3-1. Graph Representation of Szwarc Truck Assignment Problem with m=2, n=2 and p=2.

2. Many additional constraints may be added that will make the problem more realistic. First among the constraints are upper bounds on flows along particular routes of the form

$$x_{ijk} \leq Q_{ijk} \qquad (3\text{-}13)$$

where Q_{ijk} is the upper bound on flow through arc x_{ijk}. A finite upper bound might reflect a capacity constraint on the usage of a route.

The most general form of the truck assignment problem to be solved using the out-of-kilter algorithm is as follows, where a is an index relating to a set of intermediate nodes that are neither sources nor sinks of commodities. This form may be called the multi-commodity truck assignment problem.

Minimize:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} b_{ijk}x_{ijk} + \sum_{i=1}^{m}\sum_{a=1}^{f}\sum_{k=1}^{p} b^*_{iak}x^*_{iak} +$$

$$\sum_{a=1}^{f}\sum_{j=1}^{n}\sum_{k=1}^{p} b^{**}_{ajk}x^{**}_{ajk} + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ji}x_{ji} \qquad (3\text{-}15)$$

subject to:

$$\sum_{j=1}^{m} x_{ijk} + \sum_{a=1}^{f} x^*_{iak} \leq S_{ik} \qquad \begin{array}{l} i=1,\ 2,\ \ldots,\ m \\ k=1,\ 2,\ \ldots,\ p \end{array} \qquad (3\text{-}16)$$

$$\sum_{j=1}^{n} x^*_{ajk} \leq V_{ak} \qquad \begin{array}{l} a=1,\ 2,\ \ldots,\ f \\ k=1,\ 2,\ \ldots,\ p \end{array} \qquad (3\text{-}17)$$

$$D_{jk} \leq \sum_{a=1}^{f} x^*_{ajk} + \sum_{i=1}^{m} x_{ijk} \qquad \begin{array}{l} j=1,\ 2,\ \ldots,\ m \\ k=1,\ 2,\ \ldots,\ p \end{array} \qquad (3\text{-}18)$$

$$\sum_{i=1}^{m} x^{**}_{iak} = \sum_{j=1}^{n} x^{*}_{ajk} \qquad \begin{array}{l} a=1, 2, \ldots, f \\ k=1, 2, \ldots, p \end{array} \qquad (3\text{-}19)$$

$$\sum_{k=1}^{p} \sum_{i=1}^{m} x_{ijk} + \sum_{k=1}^{p} \sum_{a=1}^{f} x^{*}_{ajk} = \sum_{i=1}^{m} x_{ji} \qquad (3\text{-}20)$$

$$j=1, 2, \ldots, n$$

$$\sum_{k=1}^{p} \sum_{j=1}^{n} x_{ijk} + \sum_{k=1}^{p} \sum_{a=1}^{f} x^{**}_{iak} = \sum_{j=1}^{n} x_{ji} \qquad (3\text{-}21)$$

$$i=1, 2, \ldots, m$$

$$x_{ijk} \leq Q_{ijk} \qquad \begin{array}{l} i=1, 2, \ldots, m \\ j=1, 2, \ldots, n \\ k=1, 2, \ldots, p \end{array} \qquad (3\text{-}22)$$

$$x^{**}_{iak} \leq Q^{**}_{iak} \qquad \begin{array}{l} i=1, 2, \ldots, m \\ a=1, 2, \ldots, f \\ k=1, 2, \ldots, p \end{array} \qquad (3\text{-}23)$$

$$x^{*}_{ajk} \leq Q^{*}_{ajk} \qquad \begin{array}{l} a=1, 2, \ldots, f \\ j=1, 2, \ldots, n \\ k=1, 2, \ldots, p \end{array} \qquad (3\text{-}24)$$

$$x_{ji} \leq Q_{ji} \qquad \begin{array}{l} j=1, 2, \ldots, n \\ i=1, 2, \ldots, m \end{array} \qquad (3\text{-}25)$$

$$x_{ijk}, x^{*}_{ajk}, x^{**}_{iak}, x_{ji} \text{ are non-negative integers} \qquad (3\text{-}25a)$$

where:

a = index relating to intermediate nodes

i = index relating to supply points

j = index relating to demand points

k = index relating to commodities

x_{ijk} = number of truck loads of commodity k sent directly from source i to demand j

x^{**}_{iak} = number of truck loads of commodity k sent from source i to intermediate point a

x^{*}_{ajk} = number of truck loads of commodity k sent from intermediate point a to demand j

x_{ji} = number of empty trucks returned from demand j directly to source i

b_{ijk} = unit cost of supplying demand for commodity k at sink j directly from source i = $c_{ijk} + t_{ik} + r_{jk}$

b^{*}_{iak} = unit cost of shipping to a as an intermediate point for commodity k from source k = $c^{*}_{iak} + t_{ik}$

b^{**}_{ajk} = unit cost of using a as an intermediate point for supplying demand for k at j = $c^{**}_{ajk} + u_{ak} + r_{jk}$

c_{ijk} = cost per truck load of commodity k from i directly to j

c^{**}_{iak} = cost per truck load of commodity k from i to a

c^{*}_{ajk} = cost per truck load of commodity k from a to j

c_{ji} = cost per empty truck from j to i

t_{ik} = cost per truck load of shipping commodity k from source i

r_{jk} = cost per truck load of receiving commodity k at demand j

u_{ak} = cost per truck load of trans-shipping commodity k at intermediate point a

S_{ik} = supply in truck loads of commodity k at source i

D_{jk} = demand in truck loads for commodity k at demand j

Q_{ijk} = upper bound on flow of k from i to j

Q^{**}_{iak} = upper bound on flow of k from i to a

Q^{*}_{ajk} = upper bound on flow of k from a to j

Q_{ji} = upper bound on flow of k from j to i

V_{ak} = upper bound on trans-shipment of commodity k at intermediate point a

Inequalities (3-16), (3-17) and (3-18) express the constraints on (respectively) the amount of each commodity shipped from i, transshipped through a, and received by j. Equations (3-19), (3-20) and (3-21) express the conservation of flow constraints that the number of truck loads that leave a point must be equal to the number of trucks that enter the point. Inequalities (3-22), (3-23), (3-24) and (3-25) restrict the flow between points to be less than a bound. The integer requirement (3-25a) is satisfied by the solution of the problem as an out-of-kilter graph.

Table 3-1 shows some typical running times for the out-of-kilter solution using randomly generated data.

Table 3-1. Computation Time for Solving Some Multicommodity Trans-Shipment Problems Using the Out-of-Kilter Algorithm*

Number of Sources	Number of Int. Nodes	Number of Sinks	Number of Commodities	Number of Arcs	Number of Nodes	Execution** Time (minutes)
5	5	5	2	205	42	0.04
10	3	3	3	285	59	0.08
15	5	2	2	304	56	0.08
7	4	4	3	288	59	0.08

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* Using IBM Share Code 3536 - "Out-of-Kilter or Transportation Problem Solver" and an IBM 7094 computer.

** Includes time to generate graph and costs.

C. An Algorithm for Constructing a Graph to Represent the Truck Assignment Problem

1. For each source i, a set of p+1 nodes is created and the nodes are named y_i, y_{i1}, ..., y_{ip}, where y_{ik} represents the k'th commodity produced at the source i, and y_i represents the i'th source.
2. Directed arcs are drawn from y_i to each of the y_{ik}. The lower and upper bounds on capacity of the arcs are zero and S_{ik}^u. The unit cost is t_{ik}.
3. For each intermediate point a, a set of capacitated p nodes $z_a = (a^1, a^2, ..., a^p)$ is constructed. The upper and lower bounds on flow through the capacitated nodes are V_{ak} and zero. The unit cost is u_{ak}.
4. For each demand j, a set of p+1 nodes is created and the nodes are named q_j, q_{j1}, ..., q_{jp}, where q_{jk} represents the k'th commodity received at demand j, and q_j represents the j'th demand. Directed arcs are drawn from each node q_{jk} to q_j. The lower and upper bounds on each arc are zero and D_{jk}. The unit cost is r_{jk}.
5. Directed arcs are drawn from each node y_{ik} to each capacitated node z_a, with upper and lower bounds Q_{iak} and zero, and unit cost C_{iak}.
6. Directed arcs are drawn from each node y_{ik} to each node q_{jk}, with upper and lower bounds Q_{ijk} and zero, and unit cost C_{ijk}.

7. Directed arcs are drawn from each capacitated node z_a to each node q_{jk}, with upper and lower bounds Q_{ajk} and zero, and unit cost C_{ajk}.
8. Directed arcs are drawn from each node q_j to each node y_i with upper and lower bounds Q_{ji} and zero, and unit cost C_{ji}.

D. Decoding the Out-of-Kilter Solution of the Graph to Obtain Routings

1. No feasible solution for the graph means no feasible solution for the problem.
2. The flow from node y_i to node y_{ik} represents the number of truck loads of k supplied at i.
3. The flow from node y_{ik} to z_a represents the number of truck loads of k shipped from i to a.
4. The flow from node y_{ik} to q_{jk} represents the number of truck loads of k shipped from i to j.
5. The flow from node z_a to q_{jk} represents the number of truck loads of k shipped from a to j.
6. The flow from node q_j to node y_i represents the number of empty trucks returned from j to i.
7. The cost of the solution to the graph is the cost of the solution to the delivery problem and is optimal.

CHAPTER IV. VEHICLE SCHEDULING FROM GIVEN LOCATIONS

Introduction

The previous chapters have been devoted to large scale "flow of goods" problems with the routing of individual vehicles among collection tasks neglected. This chapter will deal with the problem of routing of individual tasks through networks for the case where the capacity of the vehicle is such that it may service more than one demand before returning to its terminal. The problems of collecting from a demand area and delivering to it are analogous and will be mentioned interchangeably. Such operations are the basic service provided by a multitude of agencies in the private and public sectors of the economy, and have attracted a great deal of analytic attention.

Distinct areas of application and solution in vehicle scheduling may be classified on the following criteria:

a. What is the capacity of the collection vehicle in relation to the demands for service?

b. Is demand continuous or discrete?

c. Is the demand for service uniform or nonuniform?

d. Are there any additional constraints besides requiring that all demand for service be satisfied by a collection vehicle that starts from and returns to a terminal?

The first main division may be made on the relationship between the capacity of the collection vehicle and the size of the demand for collection services. In this chapter, it is assumed that the service vehicle capacity is greater than an individual service demand. If it is also greater than the sum of all demand, the vehicle may visit all demand locations before returning to its terminal. These problems are referred to as single route problems, since only one route need be fashioned. If the sum of demand is such that the collection vehicle must return to its terminal before collection is completed, and then return to service additional tasks, the problem becomes a multi-route problem.

Referring to demand as discrete or continuous is done within the context of a network framework. Discrete demand implies demand located at the nodes of a network with travel between the nodes taking place along arcs. Continuous demand implies that demand is spread along the arcs of the network and is satisfied for any arc by having the collection vehicle move along that arc. An example of a discrete case might be routing problems for the delivery of petroleum products to gas stations, while the continuous case is illustrated by snow removal on streets or the delivery of mail. Uniform demand means that the demand for service at any point is the same as any other point such as would occur in snowplowing or with reasonable accuracy in residential solid waste collection. Non-uniform demand could be exemplified by the service stations, each of which could require different quantities of product.

Turning to classifications of problems, discrete, uniform problems fall into an area of scheduling theory known as traveling salesman problems, and research on these problems has been widespread. In the traveling salesman problem, a route or schedule must be made up which includes all demand locations once and returns to the starting point while minimizing the distance traveled. The term traveling salesman problem is generally applied only to the single route problem. while multi-route problems are called m salesman traveling salesman problems, where m is the number of routes to be fashioned. Since demand is uniform, a knowledge of the capacity of the salesman or service vehicle and the number of demand points to be visited, uniquely determines the number of routes.

If the demand is discrete but nonuniform, the single route problem may still be approached as a traveling salesman problem. The multi-route problem, however, becomes more complex since the number of routes to be fashioned depends on how the various demands may be agglomerated into individual route assignments. Dividing the total demand by the capacity of the vehicle gives only a lower bound on the number of routes since it may be necessary because of the different size of demand to return to the terminal with a less than full load. This problem has been discussed in the literature in detail under many different problem names, such as the delivery problem, the cloverleaf problem, the farmer's daughter problem, the vehicle scheduling problem, the truck dispatching problem and the truck routing problem. All describe roughly the same problem, and since most recent work refers to the truck dispatching problem, this term will be used as a general problem reference. Some authors have extended this problem to consider not only that all demands must be served, but that each demand must be serviced within a particular time period or in a certain order. With the addition of such constraints, the single route problem as well as the multi-route problems may no longer be approached directly as traveling salesman problems. Most of the work done for the case without additional constraints is adaptable with modification to this more difficult problem.

If the nature of the demand is uniform and continuous, some interesting techniques from graph theory may be brought to bear. For single route problems, the chinese postman problem describes the problem of traveling all arcs of a network at least once in a continuous tour and minimizing the total distance traveled. The problem is made more difficult if some of the arcs in the network are directed (one-way) rather than nondirected (two-way). No evidence of work on multi-route problems in this area has been found, although it would appear to have great application for many municipal services.

Turning to the solid waste collection problem, it is obvious that because of the size and complexity of demand it is necessary to deal with multi-route problems. Arguments may be made as to whether additional constraints are applicable. In the next section, a literature search of work done in all these areas is presented to give a feeling for the state of the art to this time. Original work by the author of this dissertation on some problem areas in m salesman problems is also presented.

A. Literature Review

1. The Traveling Salesman Problem.

This is the name given to the following problem: A salesman is to visit a group of N cities and the distances from each city to every other city are known. Find the tour that the salesman should follow that will allow him to visit all the cities and return to his starting point while minimizing the total distance traveled.

Implicit in the problem formulation is the assumption that this is a single route problem. This means that the capacity of the salesman in terms of the number of cities he may visit without returning to the starting point is greater than, or equal to N. If this is not true, the problem becomes a multi-route problem and is called the m salesman problem which is defined in the following manner: A terminal from which a salesman starts and finishes his tours, and a group of N cities with known intercity distances are given. Assume that the salesman can only visit k cities (k less than N) at one time before he must return to the terminal. Find the m tours that the salesman must make so that he may visit all the cities and minimize the distance traveled. Each tour must be of exactly length k plus the terminal, so m is uniquely determined as N divided by k. If the result of the division is non-integer, dummy cities must be added or real cities dropped in order to make m integer.

By assuming that the salesman must visit exactly k cities on each tour, all considerations of the demand at that city have been taken care of and the possibility of making more than m tours by visiting less than k cities on a tour has been ignored.

The literature is full of research on the traveling salesman problem because it contains the elements of so many scheduling problems in such diverse areas as job shop scheduling, aircraft routing, vehicle routing and production planning, but little has been written about the m salesman problem. Since most solid waste collection problems are multi-route problems, particular attention will be paid in this section to those authors' work on the traveling salesman problem that might be extended to the m salesman problem.

The traveling salesman problem, while being rather simple to formulate, has been quite difficult to solve. This is because of the large number of feasible solutions that exist for even a moderate sized problem. Consider a 20-city traveling salesman problem. For a nonsymmetric[1/] distance matrix there are 19 factorial possible ways that the 20 cities may be ordered on the route. This means that there are 1.2165×10^{17} possible feasible solutions, which can be appreciated as being a very large number by noting that if one were able on a computer to enumerate one million of these solutions per minute, it would take over 27,500 years to search all feasible solutions. Thus attention has turned towards analytic methods for finding solutions to the problem. A definitive and complete review of the theoretical development and computational experience with

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[1/] In a nonsymmetric distance matrix, the distance from city i to city j is not necessarily the same as the distance from city j to city i.

the traveling salesman problem was presented by Bellmore and Nemhauser (1968). They included a complete bibliography and discussion of the literature devoted to the subject. Those interested in following the development of the traveling salesman problem in greater detail are directed to that article. In discussing different approaches to the problem, Bellmore and Nemhauser distinguished between three basic procedures. These are tour to tour improvement, tour building, and subtour elimination. So far, all tour to tour improvement methods are heuristics[2/] in which rearrangements of the ordering of cities on the tour are made until the analyst is satisfied that the ordering obtained is a good solution. At all times the tour under consideration is feasible in that each city appears only once. Switches in ordering of cities that will improve the tour are made, and termination occurs when it appears that enough time has been spent searching or no other improvement seems apparent. Of course, such methods do not guarantee optimal results, but they do have the feature of being quite fast.

Tour building involves choosing a starting city and then electing which city to visit next. The procedure terminates when a tour returning to the original city and including all others has

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[2/] In discussing these methods, the terms heuristic solution and optimal solution will be used. A heuristic solution is one that is feasible and that is found by using a set of rules that insure that it is a "good" solution but there is no assurance that the solution found is the best, or optimal. Optimal solution implies that the procedure, if followed to completion, guarantees that the answer found is a best solution.

been generated. An example of a heuristic method using this technique is the nearest city rule, which uses as a criterion for selecting the next city to be visited, the unvisited city which is closest. Tour building has also been used in optimal procedures, such as one developed by Little, Murty, Sweeny and Karel (1963). Of particular interest is a technique developed by Pierce and Hatfield (1966) to deal with a production sequencing problem which has additional constraints dealing with job deadlines. The algorithm is based on solving the traveling salesman problem using a tour building optimization technique. Reference is made later in this literature review to extensions of Pierce's work to single and multi-route truck dispatching problems. Held and Karp (1962) developed a dynamic programming algorithm for the optimal solution of the traveling salesman problem that is limited to about 15 cities by storage requirements on present day computers. Some aspects of Held and Karp's tour building scheme are also applicable to tour building for the m salesman problem but the matter has not been pursued in great detail. Such tour building techniques also always work with feasible solutions. This is not true for subtour elimination methods, where a less constrained problem is solved which may or may not give feasible solutions. If the solution to the traveling salesman problem is feasible, it is also optimal. However, it may be infeasible in the sense that all cities are visited but several disconnected subtours, instead of one tour, are found. The solution is then sought by adding constraints to force the elimination of subtours.

A later section of this dissertation presents a subtour elimination scheme for approaching optimal solution of the m salesman problem.

2. Single and Multi-Route Truck Dispatching Problems. As noted in the preceding section, the problem to be dealt with here has been at various times referred to as the truck dispatching problem, the vehicle scheduling problem, the delivery problem, the cloverleaf problem, the farmer's daughter problem and the truck routing problem. Briefly, it deals with the scheduling of vehicles of a given capacity among discrete, nonuniform demands for the services afforded by the vehicles. Since demand is nonuniform, the number of routes to be fashioned is one of the unknowns. In addition to constraints that demand for service must be met, additional constraints on optional deliveries, earliest and latest arrival times and vehicle types are also added by some authors.

The earliest description of the problem occurs in a paper by Garvin et al. (1957). In an article on operations research in various aspects of the oil refinery industry, the authors touched on the large scale problems discussed in Chapter III, and also mentioned the problem of routing of individual vehicles. Motivated by the similarity between the vehicle scheduling problem and the traveling salesman problem, they named the problem the farmer's daughter problem, since the salesman visited certain nodes more than once and that presumably was where the daughter was. No attempt at formulation or solution was reported.

Dantzig and Ramser (1959) discussed a problem they called the truck dispatching problem. The problem they defined is as follows: Given N points or cities each with a demand for deliveries of q_i, and a terminal point that does not have a demand. Let C = the capacity of the vehicle, and in this problem

$$\max \quad q_i < C < \sum_{i=1}^{N} q_i$$

Further, a distance matrix $D=d_{ij}$ which shows the distance from any city i, or the terminal to any city, is known. The matrix is symmetric (i.e., $d_{ij}=d_{ji}$). Find the routing for the vehicle that will satisfy the demand for deliveries and the capacity constraint on the vehicle while minimizing the distance traveled.

The authors were unable to provide a model formulation that would solve the problem optimally, but presented an interesting heuristic which forms the basis for solution of large scale truck dispatching problems to this day. This is done by investigating the problem where it is desired to aggregate the various demands into pairs. This means that for every city i, the vehicle must make either the trip from the terminal to the city or the return trip from the city to the terminal. Since the distance matrix is symmetric, the distance traveled by the vehicle between the terminal p, and the cities is a constant found by calculating

$$\sum_{i=1}^{N} d_{si}.$$

The problem of minimizing the total distance traveled then reduces to the problem of minimizing only the distance traveled between cities. This may be thought of as finding the pairing of cities that minimizes the distance between cities of each pair.

Define $x_{ij} = 1$ if city i and city j are paired
$= 0$ otherwise

Then by solving

$$\text{Minimize} \quad \sum_{i=1}^{N} \sum_{j=1}^{N} d_{ij}x_{ij} \qquad (4\text{-}1)$$

$$\text{subject to:} \quad \sum_{i=1}^{N} x_{ij} = 1 \qquad j=1, 2, \ldots, N \qquad (4\text{-}2)$$

$$x_{ij} = x_{ji} \qquad \begin{array}{l} i=1, 2, \ldots, N \\ j=1, 2, \ldots, N \end{array} \qquad (4\text{-}3)$$

$$x_{ij} = (0, 1) \qquad \begin{array}{l} i=1, 2, \ldots, N \\ j=1, 2, \ldots, N \end{array} \qquad (4\text{-}3a)$$

The set of resulting x_{ij} variables gives the desired pairs and consequently the routing for the two city problem. In order to conserve time to solve the integer programming problem, the authors suggested solving the problem with constraint (4-3a) replaced by $0 \leq x_{ij} \leq 1$, which may yield fractional solutions in some cases. In that case the solution is rounded to an integer solution. The problem has been called the matching problem, and may be solved optimally very quickly, using an algorithm by Edmonds (1965). The authors' solution for larger problems is to consider the pairs formed at this iteration as a single point for the next iteration. In this manner routes are built up until the capacity of the vehicle is met. At each stage the authors used a simple exchange technique to switch pairings if any improvement might be found. However, once two cities have been paired at one stage of aggregation, they will never be separated at a future level of investigation. The authors noted that the method is heuristic, and presented computational experience for a 12 city problem for which the method does not find what they suspected to be the optimal solution.

Clarke and Wright (1964) presented a heuristic method based on Dantzig and Ramser's technique, but one which will allow vehicles with different capacities. Their method varies in the means for agglomerating the subparts of the problem and in the means for searching for possible switches at each level. Using the procedure they developed, they found a better feasible solution to the Dantzig and Ramser trial problem but could not prove that it is optimal. Their work serves as the basis for the main computerized vehicle scheduling package available from IBM as a software service to its users. This package, "System/360 Vehicle Scheduling Program", Anon. (1968), available for running on the IBM series 360 computers has two main parts, a network analysis section and a schedule producing section. The network analysis is designed to find travel distance and travel times between potential delivery points and to do some preliminary sorting to eliminate obviously dominated schedules. Input data may be in the form of coordinates or distances and the location of barriers and congested areas may be specified as well. The algorithm used for finding shortest paths through the network is

Mills' (1966) work in the decomposition properties of shortest paths problems in large networks. The scheduling procedure has additional heuristics to allow the inclusion of constraints on such factors as priority ratings, earliest and latest possible starting times, several commodities carried by one vehicle and different vehicle capacities. To date this program is the best large scale analytical tool available for analysis of vehicle scheduling problems.

Balinski and Quandt (1964) presented a formulation for a problem they called the delivery problem, which may be used to solve small multi-route truck dispatching problems. They formulated the problem as a set covering problem which requires that feasible single vehicle schedules be enumerated and then chooses from among these schedules an optimal set meeting the requirement that all demands be satisfied. For a large problem, this would require that a great many schedules be generated first before the optimization technique could be applied. However, if some intuition is available or the problem is highly constrained so that the number of feasible routes may be reduced, the method offers some appeal. The problem statement is as follows:

Minimize:
$$\sum_{j=1}^{n} c_j x_j \tag{4-4}$$

subject to the constraints:
$$\sum_{j=1}^{n} A_j x_j = E \tag{4-5}$$

$$x_j = (0,\ 1) \tag{4-5a}$$

where A_j is an m by 1 column vector representing the j'th feasible vehicle route, and whose elements

a_{ij} = 1 if the i'th city is visited on the j'th route
= 0 otherwise,

and

c_j = cost of the j'th feasible route
x_j = 1 if the j'th route is used
= 0 otherwise
E = an m by 1 column vector of 1's
n = number of feasible routes
m = number of cities to be visited.

Each feasible route is represented as a column vector with a one in the i'th position of the vector if the route visits the i'th city and a zero if it does not. It is desired to find the set of x_j's that minimize cost and satisfy the requirement that each city be visited exactly once. Running the problem as a linear programming problem without integer constraints on the variables does not usually yield integer solutions. The authors suggested obtaining a solution using an integer programming cutting plane algorithm. They report computational experience with a problem of 15 cities and 388 feasible routes. The feasible routes were examined for dominance[3] and reduced to 270. Optimal solution was obtained in 23 iterations, but a similar problem ran 200 iterations without solution.

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[3] One route dominates another if both deliver to the same cities and one has a lower cost than the other.

The idea of using a two phase algorithm in which the first phase involves generating a set of feasible solutions and the second phase is used to select from among that set the optimal routing is quite intriguing. Some form of branch and bound or selective search of the feasible solutions will allow the discarding of many dominated feasible solutions before the second phase begins. As constraints on the charcter of the routes become more demanding, the process of searching for feasible routes in phase one may even be speeded up, since many branches may be terminated early and fewer feasible solutions become candidates for phase two. What is needed is a better algorithm for solving the set covering problems of phase two, rather than cutting plane algorithms, and the work of Pierce provides a substantial step in this direction.

Pierce has written on several aspects of scheduling and truck dispatching, as well as on the development of combinatorial programming algorithms for solving set covering problems. As noted earlier, Pierce and Hatfield (1966) made use of a traveling salesman problem to look at production problems with additional time constraints. This led to Pierce's (1967) major work on truck dispatching. In the first part of this work the main concern is with single route problems with additional constraints that include earliest and latest arrival times at a specific point, optional deliveries, split deliveries and constraints on the service vehicles such as volume limit, maximum number of stops and maximum time per trip. The solution methods suggested by Pierce do not use a set covering approach but are branch and bound tour building techniques. The basic element on which branching takes place is whether or not a particular arc leading from one demand to another is used in the solution, and lower bound estimates are found using an assignment procedure similar to that of Little et al. (1963). Two ways for searching the branches are suggested. One is "flooding", in which many branches are started at once, and the choice of which branch to investigate next depends on which seems the most promising at that time. The other is to follow one branch to completion (i.e., either infeasibility, a new feasible solution, or dominance by an existing feasible solution). Then backtracking may take place up the branch until it is exhausted and another branch is started. Since the problem is characterized by extremely long computation time to find and verify optimal solutions, Pierce was more interested in the techniques that generate feasible solutions early so that the procedure may be terminated and the best feasible solution used as a heuristic. For this purpose, he feels that on the average the second technique will yield feasible solutions sooner.

In a later paper (Pierce, 1968) the author developed a combinatorial algorithm for solving general set covering problems and chose as his trial problems a set of truck dispatching problems. Computational results based on using the algorithm for different size problems are shown in Table 4-1.

Pierce also found that in dealing with the largest of his examples, problem 13, the first feasible solution was reached in

Table 4-1. Computational Experience on Truck Dispatching Problems of Various Sizes Reported by Pierce (1968)

Problem No.	Problem Size (m*n)*	Solution Time in Seconds*** for Different Forms of the Algorithm		
		Alg 1	Alg 2	IPM2
1	5 x 31	0.050	0.050	1.867
2	6 x 62	0.167	0.117	0.367
3	8 x 92	0.567	0.200	2.067
4	13 x 91	35.167	6.367	4.933
5	11 x 231	9.183	1.383	---
6	11 x 561	12.917	2.867	---
7	11 x 1023	27.267	14.383	---
8	11 x 1485	34.950	19.317	---
9	12 x 298	89.133	3.500	---
10	12 x 538	131.033	7.117	---
11	12 x 793	77.767	4.567	---
12	15 x 575	600.000**	69.483	---
13	19 x 1159	---	2400.000**	---

*m = number of cities, n = number of feasible routes

**Termination without proving optimality

***Using the IBM 7094

5.617 seconds, and the best feasible solution at termination had been found quite early (in 349.767 seconds). In a new work on improvements to the combinatorial algorithm, for which only an abstract has been circulated, Pierce and Lasky (1969) indicate that they have found an optimal solution to problem 13 in one-half minute. This is a considerable time saving, and makes an optimal procedure quite competitive with the heuristic solutions for moderate size problems for the first time.

Pierce (1969) is continuing work on solving multi-route problems both by tour building branch and bound algorithms, and by two phase, set covering methods. Garfinkel and Nemhauser (1969) have worked on a problem of political redistricting using a two phase, set covering approach as well.

Andrew and Hamann (1967) also suggested a tour generating branch and bound technique. Their work is based on the work of Little et al. (1963), and they branch on whether a particular arc is included or excluded from the tour. A single source for starting the vehicles, unequal demands at cities, and route capacities (i.e., vehicle capacities) that do not have to be equal, are features of the model formulation. A bound on each branch is computed by searching the remainder of the possible arcs which have not yet been branched upon. If a route exists which is below capacity but to which no additional load may be added, a branch must be put in back to the starting point. If some branches exist that could possibly put a route over capacity, they are eliminated as well. Then a lower bound may be estimated by using an assignment procedure which makes branch assignments based on minimum cost, but may produce subtours.

The authors reported a great deal of difficulty in convergence to optimality. They attempted to solve Dantzig and Ramser's trial problem and found the same solution as Clarke and Wright did after 560 iterations, but could not show that it was optimal. They terminated after 47,000 iterations without having finished the total implicit enumeration. At 14 seconds per thousand iterations, over 10 minutes were spent without having proved optimality on a 12 city problem. Since a very good solution was developed early, the authors suggested that the procedure be terminated early and the best solution found be used as a heuristic solution.

Hausman and Gilmour (1967) dealt with a complicated heuristic based on solving a traveling salesman problem. They considered a single source and multi-period demands. Their formulation is as follows:

$$\text{Minimize} \quad \sum_{j=1}^{n} C_1 \sum_{i=1}^{m} Z_{ij} + C_2 D_j \quad x \left\{ \max_{i \in s_j} \; f_{ij} \right\} \tag{4-6}$$

where

$$D_j = \min_{W_{ik}} \sum_{i \in s_j} \sum_{k \in s_j} W_{ik} d_{ik} \tag{4-7}$$

$$\text{subject to:} \quad Z_{oj} = 1 \qquad j=1, 2, \ldots., n \tag{4-8}$$

$$\sum_{j=1}^{n} Z_{ij} = 1 \qquad i=1, 2, \ldots., m \tag{4-9}$$

$$\sum_{i=1}^{m} W_{ik} = 1 \qquad k=1, 2, \ldots, m \qquad (4\text{-}10)$$

$$\sum_{k=1}^{m} W_{ok} = n \qquad (4\text{-}11)$$

$$\sum_{i=1}^{m} W_{io} = n \qquad (4\text{-}12)$$

$$W_{ii} = 0 \qquad i=1, 2, \ldots, m \qquad (4\text{-}13)$$

$$Z_{ij}, W_{ij} \text{ integers}; \quad Z_{ij}, W_{ik} \geq 0 \qquad (4\text{-}13a)$$

$$\sum_{n \in s} \sum_{g \in j} W_{ng} \geq 1 \quad \text{all } s, \text{ all } j \text{ (tour requirements)} \qquad (4\text{-}14)$$

where:

S_j = set of all customers in group j

s = any subset of S_j; $s' = S_j - s$

i = a customer; 0 is the terminal

f_i = frequency of delivery required by customer i

d_{ik} = distance from customer i to customer k

j = a group of customers, or route; $j=1, 2, \ldots, n$

n = number of groups (a decision variable)

C_1 = cost per customer delivery

C_2 = cost per mile of truck travel

Z_{ij} = 1, if customer i assigned to group j

= 0, if not

m = number of customers

W_{ik} = 1, if truck travels directly from customer i to customer k

= 0, if not

D_j = optimal (minimum) distance to serve group j.

The problem requires in Equation (4-7) that for each group of customers considered the minimum cost route for a vehicle be found. This is done using the traveling salesman algorithm developed by Held and Karp (1962). For the formulation only the cost of the traveling salesman solution and not the actual routing is required, and the Held and Karp algorithm can produce the cost of the route quickly even though finding the route itself takes longer. Equations (4-8) and (4-9) require that all customers be assigned to a group and the terminal be assigned to all groups. Equations (4-10) and (4-11) state that a customer may be visited from only one other customer, and the vehicle leaving it may go to only one other customer. Sending a vehicle back to one self is prohibited in (4-13) and the assignment of flow from and to the terminal must be equal to the number of groups (4-12) and (4-11). Inequality (4-14) is a tour requirement that states that for any partition of the customers, there must be at least one trip of the vehicle between the two sets. This requirement is also used in some traveling salesman problems to restrict subtours. For an N city problem there must be 2^N-1 such constraints.

To solve the problem the authors generated possible groupings of the customers, calculated the value of D_j for each group j using the Held and Karp procedure, and evaluated the solution. Then some of the groups were shifted and a new solution found. The algorithm may be terminated at any point and is obviously heuristic. Computational time for a 120 customer (city), fifteen group (route) problem was reported to be 30 minutes on a Control Data Corporation 1604, and there is no way of measuring how good the solution is.

Hayes (1967) took a different tack in developing a tour to tour improvement heuristic method where the assignments are generated randomly from a weighted probability distribution. The weighting for each demand location is based on its demand for service, its distance from the terminal and from other demands and a random element. These weightings are then used in grouping customers on routes, and heuristic traveling salesman procedure is used to order the demand in each group. The author suggested that since the above procedure takes very little time it might be repeated for a great number of trials and the best solution kept. He found the conjectured optimum solution of the Dantzig and Ramser 12 city problem in 14 out of 40 runs of the algorithm (35 percent of the time).

The only attempts found for an analytical approach to the routing of solid waste collection vehicles were by Coyle (1967) and Coyle and Martin (1967). They developed a heuristic method for looking at the overall collection frequency selection and assignment of collection areas to crews for given manpower and vehicle allocations. The routing procedure is based on dividing a city into small areas, and then agglomerating them into collection assignments based not directly on the distance between them, but on some indirect measure of the compactness of the area. A final collection area assignment for a crew per work period does not specify an order in which it is to be done. The method used for grouping the small areas into work assignments is Hess and Weaver's (1965) work on political redistricting which is itself a heuristic. The authors used this technique to generate several feasible solutions and then looked for local improvements by switching some assignments. They have dealt with a fairly large case of an English city of 150,000 population and showed a $25,000 per year decrease in collection costs for the solution generated over the present method of collection.

3. A Note About the Truck Dispatching Problem. All of the authors who have written about the problem to date characterize it as an easy problem to formulate and a difficult one to solve optimally. For this reason, many heuristics have been developed, some of which are based on optimization techniques and some of which are not. Spitzer (1969) in an article directed towards the practitioners in scheduling, raised the question of using heuristics to find workable solutions versus the use of optimization techniques to find optimal answers. He pointed out that real world problems may often have advantages over theoretical problems in that they may naturally lead to some form of partitioning or decomposition. While the first thing taught in a basic systems course is that the best solution for

a subsystem is not necessarily the best solution for the system as a whole, such approximations might be made along geographic lines. Due to the exponential increase in computation time as the number of cities increase, it is much easier to analyze three 10 city problems than one 30 city problem. Hayes (1967) suggested that the best use of optimization techniques may be to test the heuristics. To this date, little has been done to show how far off a feasible solution generated by a knowledgeable operator of a complex vehicle system is from the optimal solution. Pierce (1967) and Andrew and Hamann (1967) both suggested that the branch and bound routines they have devised to find optimal solutions are so slow that they should be used as heuristics by stopping the procedure at some specified time and using the best feasible solution to date.

4. The Chinese Postman Problem. Turning now to the problems of finding routes for vehicles in networks where demand for service is uniform and continuous along every arc in the network, some early work in graph theory paves the way for some interesting analytical approaches. In 1763, the famous Euler was given the problem of routing a parade through streets of his native city of Königsberg so that it would cross all seven bridges in the city only once. Investigating the problem, Euler found that because of the configuration of the street network and the location of the bridges, the task was impossible, and in the process postulated some general rules about finding routes that must cover all arcs of a network. An euler tour is defined as a tour that is continuous, covers every

arc in a network exactly once and returns to the starting point. In an undirected network,[4/] simple rules exist for checking to see if an euler tour exists or not. A node is even if the number of arcs that touch it is even. An euler tour exists if all of the nodes are even. If some of the nodes are odd then some arcs will have to be traced more than once. In a directed network where all the arcs may have travel limited to only one direction, similarly an euler tour exists if every node is pseudo-symmetric, or the number of arcs directed towards the node is equal to the number of arcs directed away from it. Rules for establishing an euler tour as well as more detailed description of the graph theory involved in the process, are found in the textbook by Berge (1966).

Finding the optimal route to be taken when an euler tour does not exist is called the chinese postman problem. In this problem, a route is to be found that minimizes the distance traveled on an arc more than once. Investigators such as Glover (1967) and Edmonds (1965.) have both suggested optimal algorithms that have been successful. Work by Edmonds' associates at the National Bureau of Standards (Santone, 1968), has been proposed for a case of solid waste truck routing in Columbus, Ohio, but the actual work has not yet been carried out.

[4/] i.e., a network where travel may be in either direction along an arc.

B. An Optimal Solution Method for the m-Salesman Traveling Salesman Problem

An algorithm for the solution of the m salesman traveling salesman problem under some special conditions is presented in this section. The problem to be solved is the following:

Define:

N = the number of cities to be visited excluding the terminals

s = the number of terminals for salesmen

k = the number of cities a salesman may visit excluding the terminals before returning to a terminal

m = the number of salesmen leaving and entering each terminal (the number of tours to be established from each terminal)

Given, N cities each with the same demand for services. The maximum number of cities which a salesman may visit is k. There are s terminals from which salesmen may begin their tours, visit exactly k cities and return to a terminal which need not be the one from which the salesman started. Each city is to be visited only once. Find the route for each salesman that satisfies all of the above constraints and minimizes the total distance traveled by all the salesmen.

To eliminate some of the computational problems it is assumed that each salesman visits exactly k cities. Thus smk=N, which implies that once s, N and k are known, and N/(k×s) is integer, then m is uniquely determined. If $N/(k*s)$ is not an integer, adjustments such as adding dummy cities or subtracting real cities are made.

In the context of the solid waste collection, this problem represents the routing of solid waste collection vehicles from transfer stations through their individual collection tasks. The traveling salesmen are solid waste collection vehicles, the terminals are transfer stations and the cities are small collection areas, each of which generates the same amount of solid waste per time period. The assumption that a vehicle leaves a terminal and does not return until it has visited exactly k cities closely resembles the actual work rules of collection vehicles within a city. Lacking any means of monitoring actual load on the vehicle during the collection process, the driver is usually given his route assignment and then learns by trial and error at which points collection should be stopped in order to dump the material. Usually a day's work assignment will consist of several collection routes so that the assumption that a vehicle does not necessarily return to the transfer station from which the route started is not inconvenient as long as the last collection route of the day puts the crew back at the place from which they started in the morning, in order to pick up their belongings and transportation home. The assumption that a truck may visit exactly k cities or collection areas implies that each collection area contributes 1/k truck loads. In turn, this load must be characterized as a load at a discrete point when in fact it is fairly uniformly spread along the street network to be followed in the collection task. This discrete assumption is less real than a continuous assumption. As noted in the literature survey, continuous problems are chinese postman problems and no results are known for multi-route chinese postman problems.

An important feature of the formulation of the above problem is that it will allow more than one terminal from which salesmen may leave and return. This situation is common to many routing problems including solid waste collection. The work of some authors in set covering solutions to truck dispatching problems, particularly Balinski and Quandt (1964) and Pierce (1967) might also be readily expanded to consider the multi-terminal case. In their work, a two phase procedure is necessary, with the first phase used for generating feasible routes, and the second phase used for solving the set covering problem necessary to choose the best set of the generated feasible routes. Phase one could be adjusted to allow the generation of routes from multiple terminals and the procedure used to solve the set covering problem in phase two, slightly modified so that multi-terminal requirements might be met as well.

C. A Sketch of the Algorithm for the Solution of the m-Salesman Traveling Salesman Problem

As described in the preceding section the problem for which a solution technique is desired is that of finding routes visiting exactly k cities for each of m salesmen operating from each of s terminals that will visit N cities exactly once and minimize the total distance traveled. A mathematical programming formulation of the above problem would indeed be cumbersome with respect to number of variables and constraints necessary to describe even a small problem. However, the mathematical formulation of a simpler problem which considers all the constraints except that on the number of cities each salesman must visit, is capable of fairly straightforward exposition. Consider the following:

Problem B.

$$\text{Minimize} \quad \sum_{i=1}^{N} \sum_{j=1}^{N} f_{ij} x_{ij} + \sum_{t=1}^{s} \sum_{c=1}^{N} (d_{ct} x^{*}_{ct} + d^{*}_{tc} x^{**}_{tc}) \qquad (4\text{-}15)$$

subject to:

$$\sum_{c=1}^{N} x^{*}_{ct} = \sum_{c=1}^{N} x^{**}_{tc} \qquad t=1, 2, \ldots., s \qquad (4\text{-}16)$$

$$\sum_{\substack{i=1 \\ i\neq j}}^{N} x_{ij} = \sum_{\substack{i=1 \\ i\neq j}}^{N} x_{ji} \qquad j=1, 2, \ldots., N \qquad (4\text{-}17)$$

$$\sum_{\substack{i=1 \\ i\neq j}}^{N} x_{ij} = 1 \qquad j=1, 2, \ldots., N \qquad (4\text{-}18)$$

$$\sum_{c=1}^{N} x_{tc}^{**} = m \qquad c=1,\ 2,\ \ldots.,\ s \qquad (4\text{-}19)$$

$$x_{ct}^{*},\ x_{tc}^{**},\ x_{ij} \quad \text{are non-negative integers} \qquad (4\text{-}19a)$$

where

x_{ij} = the number of salesmen who travel from city i to city j

x_{ct}^{*} = the number of salesmen who travel from city c to terminal t

x_{tc}^{**} = the number of salesmen who travel from terminal t to city c

f_{ij} = distance from city i to city j $(d_{ii} = \infty)$

d_{ct} = distance from city c to terminal t

d_{tc}^{*} = distance from terminal t to city c

m = number of salesmen dispatched from each terminal

N = number of cities

s = number of terminals

Expression (4-15) is the objective function which is the minimization of the distance traveled by the salesmen. Equations (4-16) and (4-17) express continuity of flow requirements for each terminal and for each city, respectively. They require that the number of salesmen entering a terminal or city must equal the number leaving it. In Equation (4-18) the requirement that exactly one salesman must visit each city is expressed, while in Equation (4-19) m salesmen must visit each terminal.

No explicit constraint has been written on the number of cities a salesman must visit before returning to a terminal. Denote as Problem A, the form of Problem B with the added constraint on the number of cities a salesman must visit. If Problem B is solved and the solution is a feasible solution for Problem A as well, then an optimal solution has been found for Problem A. If not, a branch and bound scheme based on solution of Problem B with added constraints will be used to find Problem A.

The constraint set of Problem B has an incidence matrix with a plus one and a minus one in each column, thus indicating it is a network problem. This insures that solutions will be integer and that some form of network algorithm may be used to find solutions in a smaller amount of time than a simplex linear programming approach. The graph is in circulation form, which means that there is continuous flow in the graph from sources to sinks and then back through directed arcs to the sources. The method of solution will be the out-of-kilter algorithm, but in order to use it some changes in the form of the problem will be made. Equations (4-18) and (4-19) express the requirements that flows through nodes representing cities must be exactly one, and through nodes representing terminals exactly m. This may be accomplished in a graph by the use of a capacitated node, as described in Chapter II.

Each city is represented by a capacitated node with the upper bound u_i, equal to the lower bound l_i, equal to one. Entering the receptor of the capacitated node are two sets of arcs: set A_i represents arcs from other cities to i, and set A_i^* represents arcs from the terminals to i. Note that $A_i \cap A_i^* = \Phi$, the empty set. Leaving the transmitter node of the capacitated node are arcs which also comprise two sets: B_i is the set of arcs which go to other cities from i, and B_i^* is the set of arcs that return to the terminals from i. $B_i \cap B_i^* = \Phi$.

In a similar manner the terminals may be represented by a capacitated node with upper bound equal to lower bound equal to the number of salesmen who must enter and leave a terminal m. The receptor of the capacitated node receives arcs from all cities and the transmitter sends arcs to all cities.

Consider the following sets of arcs:

P = (the set of arcs that makes up the capacitated nodes representing cities)

T = (the set of arcs that makes up the capacitated nodes representing terminals)

Then the problem may be shown in simple circulation form as:

$$\text{Minimize} \quad \sum_{i=1}^{P} C_i X_i$$

$$\text{subject to:} \quad \sum_{i \in B_{(k)}} X_i = \sum_{j \in A_{(k)}} X_j \qquad k=1, 2, \ldots., a \qquad (4\text{-}20)$$

$$0 \leq X_i \leq 1 \qquad i \notin P \cup T \qquad (4\text{-}21)$$

$$X_i = 1 \qquad i \in P \qquad (4\text{-}22)$$

$$X_i = m \qquad i \in T \qquad (4\text{-}23)$$

$$X_i = \text{non-negative integer} \qquad (4\text{-}23a)$$

where X_i = flow in arc i

p = number of arcs

a = number of nodes

C_i = unit cost of using arc i

$= 0$ if $i \in P$

$= 0$ if $i \in T$

$= f_{ij}$ if i is an arc joining cities i and j

$= d_{ct}$ if i is an arc joining city c and terminal t

$= d^{*}_{tc}$ if i is an arc joining terminal t and city c (where these symbols have been defined previously)

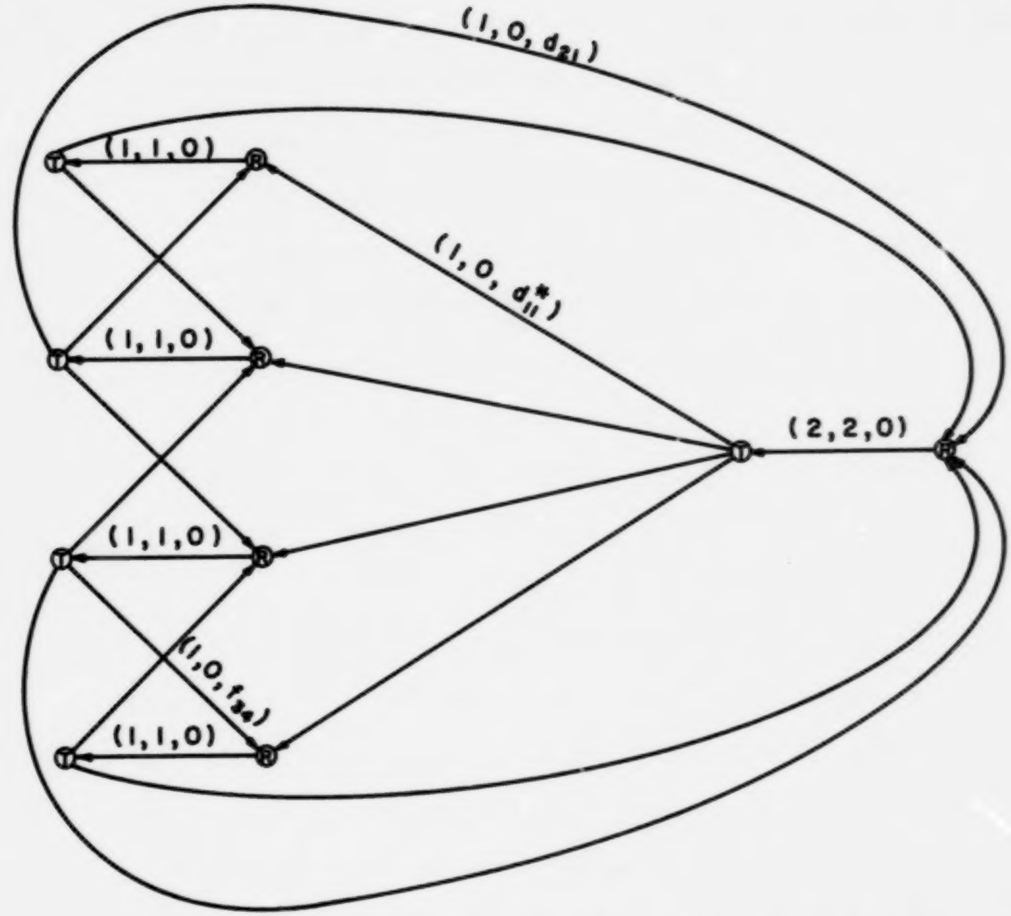
$A_{(k)}$ = arcs leaving node k

$B_{(k)}$ = arcs entering node k

m = number of salesmen

An example of the graph representing this formulation is shown in Figure 4-1 for the case of N=4 cities, m=2 salesmen per terminal, s=1 terminal and k=2 cities to be visited by each salesman. As indicated earlier, such a problem could be solved by a matching algorithm instead of the techniques under discussion now. However, this case is presented here to give an example for a graph that is easily drawn and understood. The number of arcs and nodes required to characterize a problem is quite large. For an N city, s terminal problem when all cities may be reached from all other cities and terminals, the number of nodes is

$$2s + 2N$$


(1,0, d_{21})
(1,1,0)
(1,0, d_{11}^{*})
T
R
(1,1,0)
(2,2,0)
(1,1,0)
(1,0, f_{34})
(1,1,0)

(UPPER BOUND, LOWER BOUND, UNIT COST)
NOT ALL POSSIBLE ARCS ARE SHOWN.

Figure 4-1. Graph for a Problem with N=4, p=1, m=2, k=2.

and the number of arcs is

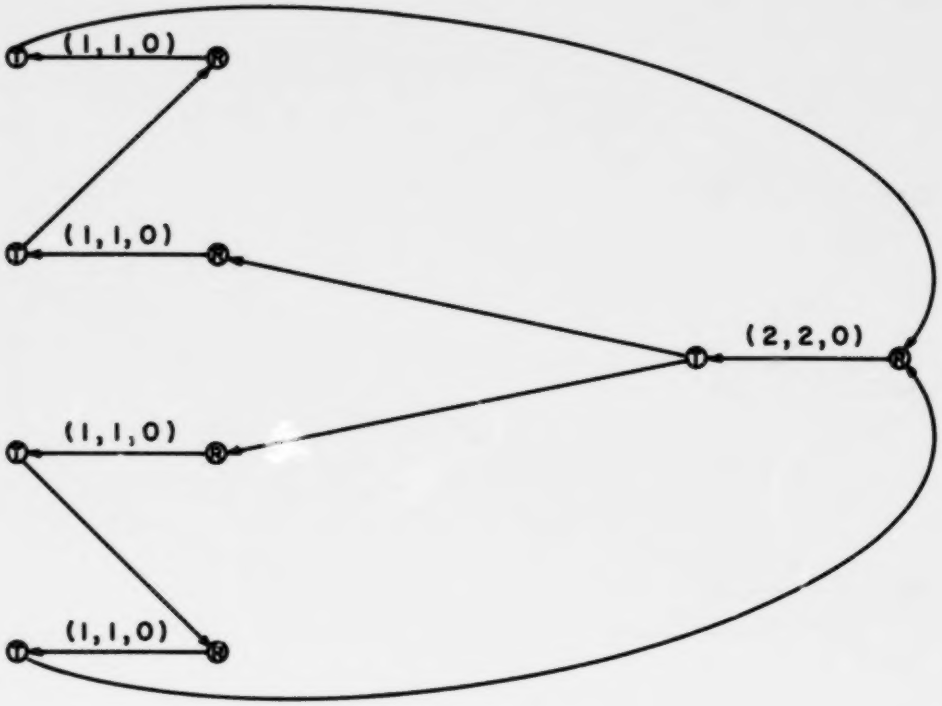
$$N^2 + 2s*N + s$$

For $N=16$, $s=2$ there are 34 nodes and 322 arcs.

The relationship of the solution of Problem B to Problem A may be summarized in the following three cases:

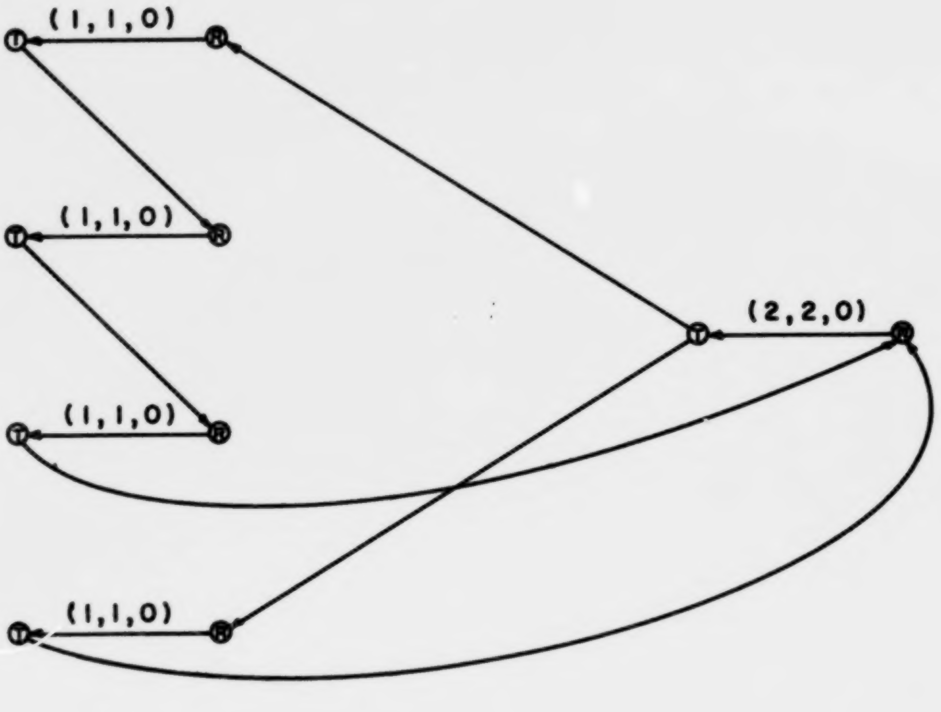
> Case 1. The solution of Problem B is also a feasible solution for Problem A. This is shown in Figure 4-2. This case will be referred to as a feasible solution.
>
> Case 2. The solution of Problem B is such that m routes are found from each of the terminals, but not all visit exactly k cities before returning. All cities are visited by a route starting from and returning to a terminal. Such a solution is shown in Figure 4-3. This will be called a regular infeasible solution.
>
> Case 3. The solution of Problem B generates more than ms routes and only ms of these start and end at a terminal. A feasible solution to Problem B might contain some routes that start at a city, visit other cities and then return to the starting city without ever touching the terminal. This will be called a phantom infeasible solution. An example is shown in Figure 4-4.

If an infeasible solution such as Case 2 or Case 3 appears, additional work will have to be done to find the optimal solution


(1,1,0)
(1,1,0)
(2,2,0)
(1,1,0)
(1,1,0)

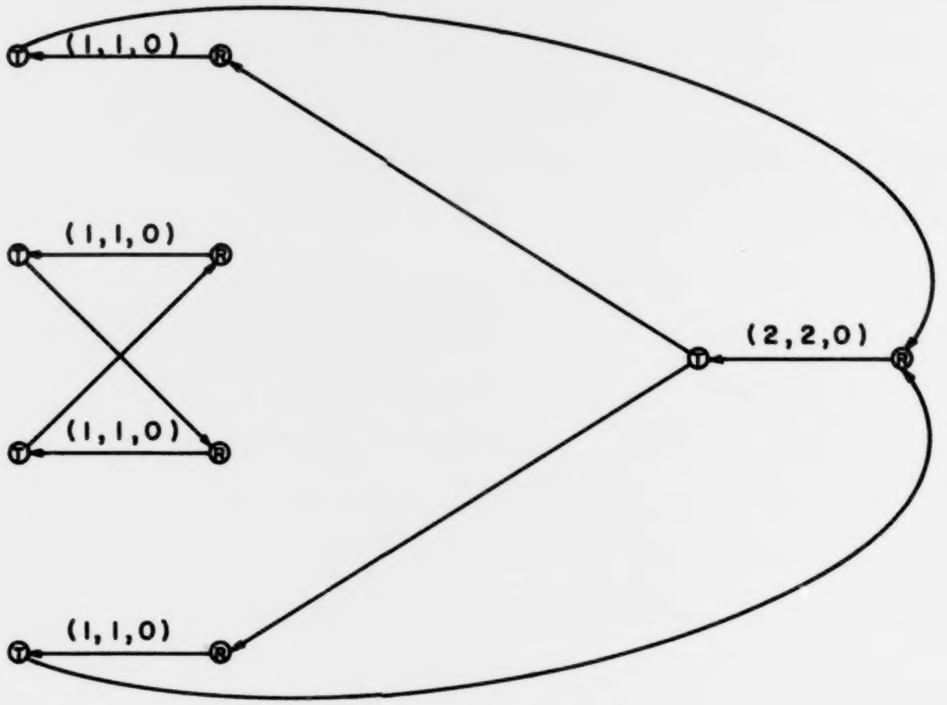
(UPPER BOUND, LOWER BOUND, UNIT COST)

Figure 4-2. Feasible Solution to Problem A.


(1,1,0)
(1,1,0)
(1,1,0)
(1,1,0)
(2,2,0)
T
R

(UPPER BOUND, LOWER BOUND, UNIT COST)

Figure 4-3. Regular Infeasible Solution to Problem A.


(1,1,0)
T
R
(1,1,0)
T
R
(1,1,0)
T
R
(1,1,0)
T
R
(2,2,0)
T
R

(UPPER BOUND, LOWER BOUND, UNIT COST)

Figure 4-4. Phantom Infeasible Solution to Problem A.

to Problem A. The type of infeasibility will suggest the way that the branch and bound procedure can be developed.

The general plan of attack will be the same as for the capacitated trans-shipment facility location algorithm. Solve Problem B. If the solution to Problem B is Case 1, it is feasible to Problem A. Then since Problem A is more highly constrained than B the optimal solution to A has been found. If the solution to B is of Case 2 or Case 3 and therefore infeasible to A, the problem will be broken into a series of subproblems which divide the set of feasible solutions to B into mutually exclusive collectively exhaustive subsets. This is done by taking some infeasible route in the problem solution and prohibiting arcs involved in that route from being in a further solution. Suppose the solution to Problem B contains a route which starts at terminal t, visits h cities, $a_1, a_2, \ldots, a_h$ and returns to terminal t^*. Then the set of feasible solutions may be divided into h + 1 subsets with the first subproblem having all arcs broken from t to all the cities a_1 through a_h. The second subproblem breaks all arcs from a_1 to all other cities (a_1 through a_h) and the terminal t^*. This is continued until the h + 1'st subproblem breaks all arcs from a_h to a_1 through a_{h-1} and to t^*. The arcs are broken by setting their unit costs to infinity. Each subproblem is solved to find its lower bound. Since in each subproblem an arc has been broken which previously had flow into it, the flow pattern will change and the cost of the new solution will be the same or higher than the previous solution. The cost of each of the subproblems is examined and the one with the least cost is found. If its solution is feasible to A, an optimal solution to A has been found. If it is not feasible to A, more subproblems are generated in the same manner and the lowest cost subproblem is again investigated. This is continued until the lowest cost subproblem is a feasible solution to Problem A.

The rules used to set up the subproblems are called break rules. Since there are two different types of infeasible routes, it is possible to have several different break rules. One break rule might be to choose for branching the smallest regular infeasible route. Another is to choose for branching the smallest phantom route and break its arcs. Experience with both break rules is reported.

D. A Detailed Algorithm for the m-Salesman Traveling Salesman Problem

1. The following data are given:

N = the number of cities to be visited excluding the terminals

s = the number of terminals

m = the number of salesmen leaving each terminal

k = the number of cities excluding the terminals that a salesman must visit before returning to a terminal

The number of salesmen, m, is an integer determined by $m = \frac{N}{(ks)}$. If this calculation is not integer, N is adjusted by adding dummy cities or subtracting real cities until m is integer.

d_{ct} = distance from city c to terminal t

d^*_{tc} = distance from terminal t to city c

f_{ij} = distance from city i to city j

These distances are not necessarily symmetric. That is, d_{ab} need not be equal to d^*_{ba} nor f_{ij} equal to f_{ji}.

2. Construct a graph in the following manner:

There will be two sets of nodes, P and T, which make up the graph.

Define P = (the set of nodes that represent cities)

For each of the N cities, a capacitated node is established. This is accomplished by representing the capacitated node by two nodes with a directed arc between them, as shown in Figure 4-2. The node that sends the directed arc is known as the receptor node, and the node that receives the directed arc will be known as the transmitting node of the capacitated node. It will be required that the flow through the capacitated node be exactly one unit. This is accomplished by setting the upper and lower bound on flow through the arc joining the receptor and transmitting nodes equal to one unit. Refer to the capacitated node representing a city as a city node.

Define T = (the set of nodes that represent terminals)

For each of the s terminals, a capacitated node is constructed in the same manner as for the cities. It will be required that the flow through each capacitated node representing each terminal be exactly m units, since m salesmen will pass through each terminal. This is accomplished by setting the upper and lower bounds on the flow through the directed arc joining the receptor and transmitting node as m units. Refer to the capacitated node representing each terminal as a terminal node.

Arcs of the graph are defined in the following manner:

From T to P: define a directed arc from the transmitting node of each terminal node to the receptor node of each city node. The unit cost on the arc is d^*_{tc} and there is an upper bound of one on the arc.

From P to T: define a directed arc from the transmitting node of each city node to the receptor node of each terminal node with a unit cost of d_{ct} and an upper bound of one.

Within P: from any city node i to any other city node j, $i \neq j$, define a directed arc with unit cost f_{ij} and an upper bound of one, from the transmitting node of i to the receptor node of j.

3. Define S = (set of all arcs of the graph that have been excluded from the solution and hence must have zero flow)

$\underline{S}$ = (set of all arcs of the graph not yet excluded from the solution)

$\underline{S}^*$ = (set of all arcs of the graph)

Then $S \cup \underline{S} = \underline{S}^*$ and $S \cap \underline{S} = \phi$, the empty set

Initialize $S = \phi$

$\underline{S} = \underline{S}^*$

4. Find the minimum cost, maximum flow through the graph using the out-of-kilter algorithm. If this is the first time step 4 has been entered and there is no feasible solution to the graph, there is no feasible solution to the problem. Define the cost of this flow as CGRAPH.

5. Examine the solution of the graph to see if it is a feasible solution to the m salesman traveling salesman problem. This is done by decoding the solution to determine the routes for each salesman. From each terminal node only m arcs

leaving it may have non-zero flow and for each city only one arc entering and one arc leaving it may have non-zero flow. The routes for each of the salesmen may then be calculated by starting at a terminal and selecting an arc to a city, say city i^*, from that terminal with non-zero flow. Then examine the city node i^*, and find the single non-zero arc leaving it. If this arc goes to a terminal the route is complete. If it goes to another city, say i^{**}, examine the city node i^{**} and apply the same rules until the route is terminated. Continue until each of the m routes at each of the s terminals has been enumerated. If each of the routes enumerated visits exactly k cities, the solution is feasible.

6. If the solution is feasible, go to step 7. If it is not, one of the infeasible routes is chosen for branching. Select the smallest route in terms of number of cities visited as the candidate for branching. In case of a tie for the smallest, choose the first enumerated.

7. Store information about the solution. File on a list, ranked on the cost CGRAPH, of all solutions generated, data about the feasibility, members of the S set and the candidate for branching.

8. Remove the first solution on the list. If that solution is feasible, it is optimal.

9. If the solution is infeasible, several subproblems are set up. Suppose the route chosen as the candidate for branching starts at terminal t, visits h cities, a_1, a_2, ..., a_h, and returns to terminal t^*. Then h + 1 subproblems will be generated. First eliminate all arcs listed in the S set by setting their costs to infinity.

9a. Subproblem 1 - Break all arcs from t to each of the cities a_1, a_2, ..., a_h, by setting the costs of these arcs to infinity and add these arcs to the S set. Repeat steps 4 through 7. Restore the arcs just broken to their original costs and go to 9b.

9b. Subproblem 2 through h + 1 - If this is the g'th subproblem, break all arcs from a_{g-1} to each of the other h-1 cities and to t^* and add these arcs to the S set. Repeat steps 4 through 7 and restore the arcs just broken and go to the next subproblem. When all h + 1 subproblems are completed, go to step 8.

E. Computational Experience

The algorithm was programmed in ASA Fortran IV and run on an IBM 7094 computer. The out-of-kilter algorithm was written by Clasen and made available through the IBM SHARE Library. The list processing subroutines used for keeping track of solutions were written by Bellmore (1966). Two possible break rules were considered for setting up subproblems. Break rule A selected the smallest infeasible route that passed through a terminal as the route for branching. Break rule B involved selecting a phantom route (Case 3 infeasibility) which by definition does not pass through a terminal, as the candidate for branching. If no phantom routes exist, rule A is employed. Both rules were tried in early runs for some 8 city problems and showed little difference. Only break rule A was used in the larger city cases. Because of dimension limitations and the use of decimal packing instead of binary packing for the storage of information about solutions, many problems were limited by exceeding storage rather than by time limits. Future attempts at working with this algorithm will require better programming techniques and more careful packing of data to insure greater efficiency, but such programming efforts were not deemed necessary for this trial demonstration of the method. Runs were made with one and two terminals for cases of 8, 12 and 16 cities. Results of these runs are shown in Table 4-1. Runs were made with symmetric and asymmetric data. The symmetric case generally took longer to solve. This same property in traveling salesman problems is explained by Bellmore and Malone (1968). The procedure has a tendency to produce very few feasible solutions before an optimum is found. This works to a considerable disadvantage in the last two runs, since considerable time was invested before termination without finding one feasible solution. The algorithm could be speeded up and have more feasible solutions generated, by taking each infeasible solution and applying some heuristic procedure to round it to a feasible solution. The development of the feasible solution would not take long, and if found to be lower than the current best feasible solution, a tighter bound on the problem has been found. At termination before optimal solution, a good feasible solution would still exist.

Edmonds and Johnson (1969) are presently working on a problem area known as degree constrained subgraphs which would appear to have application to the above problem. Basically, the problem is to minimize the cost of using arcs subject to constraints that a specified number of arcs must enter and leave each node. This could form the basis for generating tours for each vehicle. The algorithms Edmonds and Johnson deal with are as fast as the matching problem algorithms. Based on Bellmore and Malone's (1968) experience, it would be expected that this would alleviate the problem of vastly different computation time for symmetric and nonsymmetric data cases.

Table 4-2. Computational Experience for Problems with Randomly Generated Data

No. of Terminals (s)	No. of Vehicles at each Terminal (m)	No. of Cities (N)	Data Type	Features of Graph		No. of Feasible Sol'ns	No. of Times OKA Called	No. of Feasible Sol'ns Found	Running Time on IBM 7094 (minutes)
				No. of Arcs	No. of Nodes				
1	2	8	ASYM***	73	18	1,680	8	2	0.03
1	4	8	ASYM	73	18	27	27	2	0.10
1	4	12	ASYM	169	26	1,360	76	1	0.48
1	2	12	ASYM	169	26	665,280	150	2	0.75
1	3	12	ASYM	169	26	11,880	101	2	0.52
1	3	12	SYM***	169	26	5,940	101	2	0.52
2	2	12	SYM	169	26	47,520	46	1	0.32
1	4	16	ASYM	289	34	43,680	251	4	1.87
2	4	16	ASYM	322	36	50,000	210	8	2.39
2	2	16	ASYM	322	36	174,720	215	3	1.87*
2	4	16	SYM	322	36	25,000	230	0	2.12**
2	5	20	ASYM	1,682	44	4.4×10^6	330	0	3.21**

*Stopped because of storage with lower bound close to a generated feasible solution.

**Stopped because of storage with no feasible solution found.

***ASYM means asymmetric data.

SYM means symmetric data.

CHAPTER V. THE ANALYSIS OF A SOLID WASTE COLLECTION SYSTEM

Introduction

In this chapter, a large scale solid waste collection and disposal system will be investigated, using some of the techniques suggested in previous chapters. The system chosen for examination was that operated by the City of Baltimore, Maryland. This selection was based on several factors. First, Baltimore is a good example of a large city with extensive investment in a public solid waste collection system. Second, a great deal of data has been collected concerning the operation of the system, and is available. Analysis of the Baltimore system has already been attempted by Truitt (1968, 1969), who used the city as an example in the development of a simulation model of collection. Through his excellent work, considerable data and insight into the system are available for use with the models proposed in this study. All data taken from Truitt's work will be referenced.

A. Characterization of the Baltimore Solid Waste Collection System

The City of Baltimore, Maryland, had a population in 1960 of just over one million, a land area of 80.3 square miles and physical characteristics that ranged from high density urban slums to verdant low density residential subdivisions. The municipal functions of solid waste collection and disposal as well as general sanitation activities are carried out by the Bureau of Sanitation of the Department of Public Works. For purposes of logistics and supervision, the city is subdivided into five autonomous districts, the northwestern, the northeastern, the central, the eastern and the western. These districts are shown on the map in Figure 5-1. The Bureau employs 1271[1/] persons and owns approximately 340 vehicles and pieces of equipment, and operates on a budget of over seven million dollars per year. A breakdown of the functions and their costs is shown in Table 5-1. As indicated in the table, collection accounts for almost 85 percent of the department's operation and the predominant collection operations are mixed refuse collection and street cleaning.

The Bureau collects mixed refuse from residential and non-commercial sites twice a week, once on either Monday, Tuesday or Wednesday, and once on either Thursday, Friday or Saturday. Since collection in the earlier part of the week represents the pick up of four days' accumulation of wastes, crew size is one driver and

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[1/] From Report of Department of Public Works (Anon, 1966a) three laborers. In the later part of the week crew size is reduced to one driver and two laborers. Two types of vehicles are generally used for collection. They are both equipped with compaction equipment and have capacities of 20 and 13 cubic yards of compacted solid wastes each. The larger vehicle is often referred to as a five ton truck and the smaller as a three ton truck. The larger vehicle is thought to be more efficient than the smaller and is assigned to the general tasks while the smaller is saved for alleys and other size restricting jobs. In 1965 the Bureau collected 336,893 tons of mixed refuse, 31,395 cubic yards of ashes and 216,622 cubic yards of street dirt. The collection of the mixed refuse from residences involves 92 vehicles, each with three daily route assignments per week for a total of 276 routes. There are two city owned incinerators for the disposal of wastes, and the solid waste collected is taken directly from the collection area to the incinerator by the collection vehicle. One of the incinerators is the Number Three (Reedbird) with a capacity of 600 tons per 24 hour day, and the other is the Number Four (Pulaski) with a capacity of 800 tons per 24 hour day. The location of these facilities is also shown in Figure 5-1.

Table 5-1. Cost of Operations and Services Provided by the Bureau of Sanitation, City of Baltimore, Maryland, 1965.

Operation	Yearly Expense	Percent of Total
Street Cleaning	2,792,000	37.2
Ash Cleaning	202,000	2.7
Mixed Refuse Collection	3,034,000	40.4
Other Miscellaneous	324,000	4.4
Collection Expenses	6,352,000	84.7
Incinerators	1,110,000	14.8
Dumps	40,000	0.5
Disposal Expenses	1,150,000	15.3
Totals	7,502,000	100.0

B. Study Objectives

Analysts involved in the study of, and planning for, large scale public systems such as the solid waste collection and disposal systems must be concerned with both long range and short range planning. Not only must they study the evolution of new technology and methods that may dramatically change alternatives and options in the future, but they must be concerned with the present on-going vital existence of the system. This study will be directed towards the analysis of the system within the short range picture, in order to give some feeling for the alternatives that are available for immediate introduction into the system. These alternatives may be physical changes such as the building of new structures, policy changes such as changes in the manner and character of the services and rules of operation of the system, or changes in the methods for carrying out certain objectives of the system. Short range alternatives that fall into each of these categories make up the prominent issues that require analytical attention.

The three questions that make up the theme of this study are:

1. Are transfer sites within the city a feasible alternative? If so, where should they be located and to what capacity should they be built?
2. What is the cost and effect of increasing collection frequency from twice a week to three times a week?
3. Under what conditions and at what price would rail haul become a feasible alternative for the city?

Subordinate to these main questions but still of great interest to the analyst and planner are such additional questions as:

4. How sensitive is the system to changes in parameters and cost estimates?
5. What are the effects of political and esthetic constraints that might force a change from the solution most economically efficient from the engineering point of view to one that is perhaps more acceptable to segments of the community?
6. Are there advantages to cooperation between governmental units within a region where such cooperation does not now exist?

These questions will be subject to some preliminary investigation in the next sections, using the large scale facility location model of Chapter II. More detailed study would also require some of the routing techniques of Chapter IV to achieve overall efficiency of operation, but these have not been included in this overview.

C. Calculations and Data for the Baltimore Study

1. Collection area to be investigated

The area chosen for study was the same area studied by Truitt (1968, 1969). It comprises the northwest division of the City of Baltimore, and a map of the area and its relationship to the city are shown in Figure 5-1. Within this area it is necessary to subdivide into smaller subareas for analysis. A convenient way to do this is to consider each census tract within the area as a subarea. Exact population and neighborhood density type, based on the 1960 census, are accurately known. There is a total of 40 census tracts within the northwest quadrant and these are shown in the map of Figure 5-3. A coordinate system has been set up with its zero point at the northwest corner. Information about the population, number of housing units, neighborhood density type and the x-y coordinates of the approximate centroid of the tract are given in Table 5-2. Neighborhood density types are based on estimates made from census data by the City of Baltimore, Department of Planning, as reported by Truitt (1968). These definitions are shown in Table 5-3. The housing unit density is important because it gives some indication of the speed of collection within a subarea. It would be suspected that the more closely spaced the pick up points are, the faster the collection process may be in terms of pounds per hour collected. Truitt, in a statistical study of the Baltimore data, shows that there is a significant difference between density

Table 5-2. Pertinent Data Concerning the Forty Census Tracts of the Northwestern Division, Baltimore, Maryland

Tract Number[*]	Neighborhood Density	Number of Household Units[**]	Population[**] 1960	Coordinates	
				x	y[***]
28-1B	1	2,482	2,482	3,750	13,000
28-2	1	2,035	5,646	3,806	9,000
28-3	1	1,194	3,546	4,000	26,000
28-1A	2	1,099	3,531	4,000	8,500
27-19	1	1,490	4,622	10,000	6,500
27-20	1	4,634	13,437	5,750	3,000
27-17	2	2,079	6,756	14,500	8,500
15-12	3	2,011	6,091	17,500	17,000
15-13	3	1,910	5,623	15,000	14,500
27-16	3	1,979	6,127	16,000	12,000
27-18	3	3,207	9,808	10,500	11,000
15-7A	2	865	2,944	14,000	22,500
15-8A	2	1,787	4,811	11,500	21,500
15-8B	2	493	1,390	11,500	25,000
15-9	2	1,574	4,950	9,000	24,500
15-10	2	2,249	6,193	9,250	16,500
15-11	2	2,691	7,677	13,500	17,000
27-11	1	780	2,786	31,500	10,500
27-12	1	2,159	6,648	31,000	3,500
27-13	1	913	3,183	25,000	4,000
27-14	1	1,543	4,398	26,500	11,000
27-15	1	1,904	6,427	20,000	6,500
12-1	2	1,887	3,495	32,000	15,000
13-5	3	523	1,762	27,500	20,500
13-6	3	1,426	4,336	27,500	18,000
13-7	3	1,918	5,429	27,500	15,000
13-8	3	2,039	6,824	22,500	15,500
12-2	4	3,839	7,499	32,000	16,500
12-3	4	1,987	5,483	33,000	22,000
12-4	4	1,687	5,469	33,000	24,500
12-6	4	1,532	3,177	32,000	22,000
12-7	4	1,474	4,705	30,000	22,500
13-1	4	1,917	4,648	27,500	23,500
13-2	4	2,062	5,859	27,500	25,000
13-3	4	1,598	4,904	23,500	20,000
13-4	4	1,154	4,026	22,500	20,500
15-4	3	1,576	5,800	20,500	25,000
15-5	3	741	2,064	18,000	22,000
15-7B	3	1,787	4,298	14,500	25,000
15-6	3	2,150	7,725	14,000	27,000

[*] Tract number is U. S. Census designation.

[**] Population and number of household units from 1960 census.

[***] Coordinates are in feet measured to the approximate centroid of the tract. Point x=0, y=0 is located at the extreme northwest corner of the Baltimore City boundary.

type 1 and density types 2, 3, and 4, but no significant differences between 2, 3, and 4. Thus only two sets of neighborhood density types need be considered, type 1 and other than type 1.

Table 5-3. Classification of Neighborhood Types

Classification	Housing Units per Acre
1	Ten or less
2	Eleven to twenty
3	Twenty-one to forty
4	More than forty

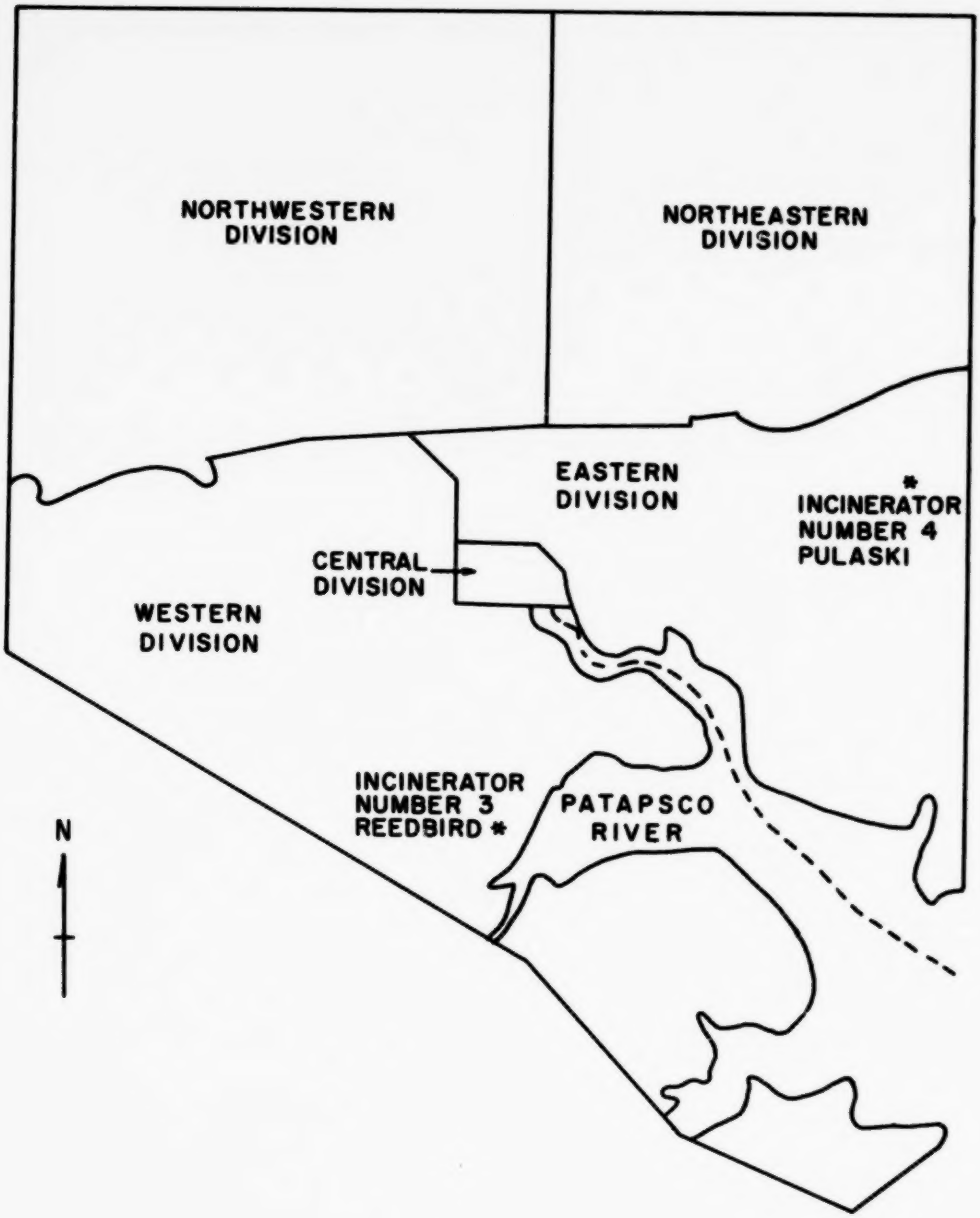
NORTHWESTERN DIVISION
NORTHEASTERN DIVISION
EASTERN DIVISION
INCINERATOR NUMBER 4 PULASKI
CENTRAL DIVISION
WESTERN DIVISION
INCINERATOR NUMBER 3 REEDBIRD
PATAPSCO RIVER
N

Figure 5-1. Operating Divisions of Bureau of Sanitation, Baltimore, Maryland.

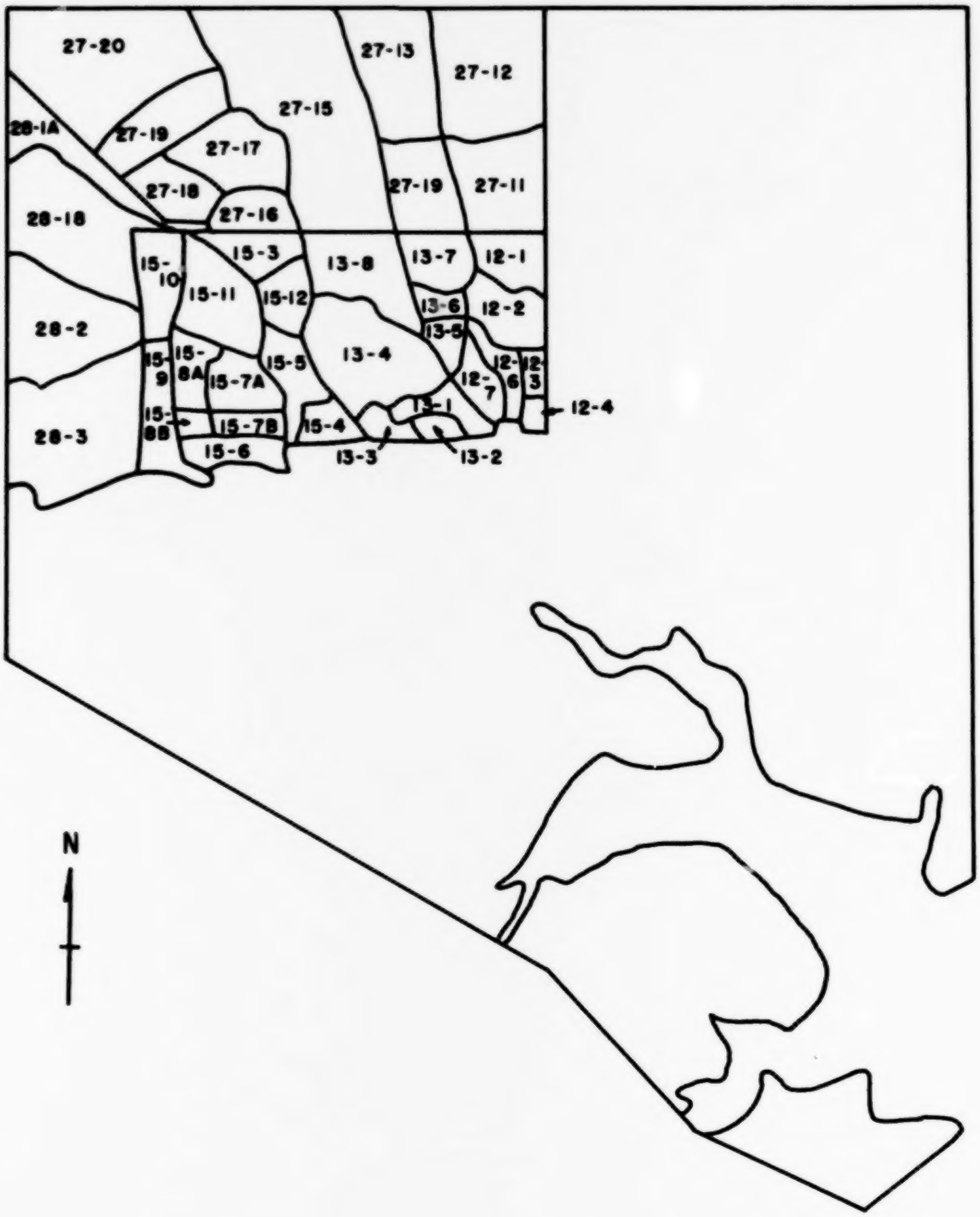
27-20
27-13
27-12
27-15
28-1A
27-19
27-17
27-18
27-19
27-11
28-1B
27-16
15-3
13-8
13-7
12-1
15-10
15-11
15-12
13-6
12-2
28-2
13-5
15-9
15-8A
15-7A
15-5
13-4
12-7
12-6
12-3
13-1
12-4
15-8B
15-7B
15-4
28-3
15-6
13-3
13-2
N

Figure 5-2. Map Showing Census Tracts in Northwestern Division of Baltimore, Maryland.

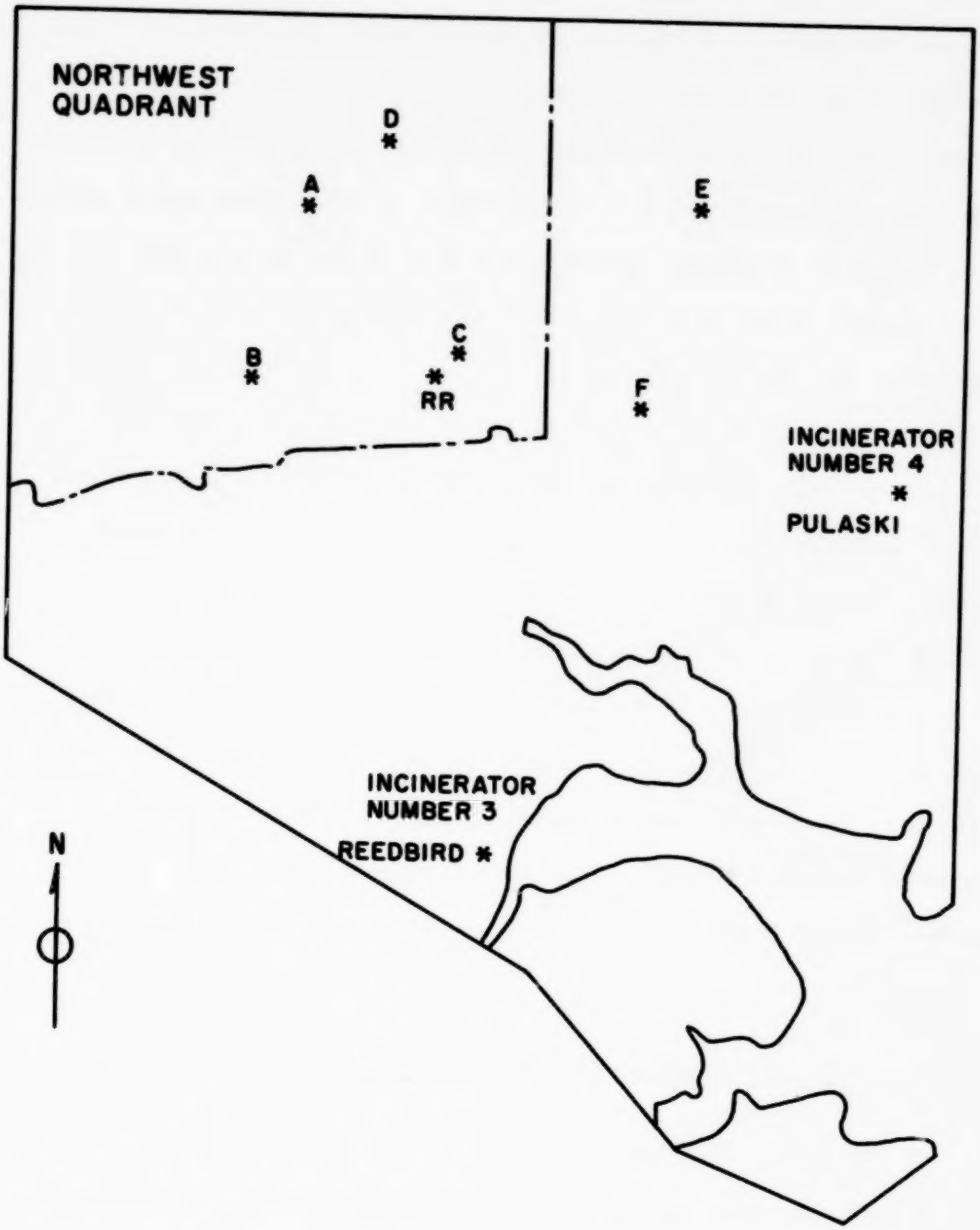
NORTHWEST
QUADRANT
D
A
E
C
B
RR
F
INCINERATOR
NUMBER 4
PULASKI
INCINERATOR
NUMBER 3
REEDBIRD
N

Figure 5-3. Map Showing Location of Proposed Transfer Sites and Present Incinerator Sites, Baltimore, Maryland.

2. Speed of collection within a subarea

The speed of collection within a subarea is a function of the neighborhood density and the number of days since the last collection. When the collection frequency is two times a week as it is in Baltimore, one collection is of four days of solid waste accumulation, and the other of three. Normally the City of Baltimore system uses a driver plus three laborers for the larger collection, and a driver plus two laborers for the smaller one.

On pages 174-176 of Truitt (1968) several histograms are presented for collection rates in pounds per hour as a function of days since last collection and neighborhood type. The histograms based on twice a week collection are computed from actual observations within the system. Estimates were made of histograms of three times a week collection frequency. In this case the first collection of the week has three days accumulation and the remaining two have two days accumulation. The number of personnel assumed is one driver and two laborers. Table 5-4 presents weighted averages of the histograms for different conditions which will be used in the analytical models.

3. Cost of collection within a subarea

Cost of collection is assumed to be a function of the collection frequency, the speed of collection, the neighborhood type and population, and the work rules assumed. The following data are pertinent to this computation:

Table 5-4. Average Collection Rate in Pounds per Hour for Different Days Since Last Collection and Neighborhood Type

Days Since Last Collection	Neighborhood Type	Average Pounds per Hour Collected
2	1	2,500
2	2, 3, 4	4,580
3	1	3,000
3	2, 3, 4	6,300
4	1	5,000
4	2, 3, 4	6,300

Table 5-5. Collection Costs in Dollars per Hour

Days Since Last Collection	Labor Necessary	Labor Cost Dollars/Hr	Vehicle Cost Dollars/Hr	Total Cost/Hr
4	1 driver + 3 laborers	12.63	4.40	16.73
3	1 driver + 2 laborers	9.33	4.40	13.73
2	1 driver + 2 laborers	9.33	4.40	13.73

a) Collection vehicle costs - The City of Baltimore estimates its cost per hour of vehicle use for the 20 cubic yard collection vehicle as $4.40 per hour.

b) Typical labor rates are twenty dollars per day for a driver and eighteen dollars per day for a laborer. The normal crew assignment for a collection of four days since last collection is one driver and three laborers per vehicle. This drops to one driver and two laborers for shorter intervals between collections.

c) To compute an hourly labor rate from the daily rate it is necessary to compute the average time spent in actual work tasks. Truitt (1968) has estimated that between 45 and 50 minutes per hour are spent in actual productive time, with the remaining time being spent in start up, repairs, off service time, etc. For the purpose of this study, 45 minutes per hour or 6 hours per day will be used as the productive time. This yields a productive hourly rate for drivers of $3.33 per hour, and for laborers of $3.00 per hour.

Table 5-5 shows labor, vehicle and total costs for different types of collection.

d) Waste load generated - The waste load generated per person per day is a statistic that shows great variation. Truitt (1968) after extensive study of Baltimore residential data, used 1.95 pounds per person per day and calculated a standard deviation of 0.09 pounds per day. This figure will be varied to show the sensitivity of the solution to it.

e) Total collection cost in a subarea per week - The total collection cost per week in a subarea is calculated in the following manner:

Define p_i = population of the i'th subarea

k_i = weight of solid waste generated per person per day with twice a week collection frequency

k_i' = weight of solid waste generated per person per day with three times a week collection frequency

l_j = labor cost per hour for a crew of one driver plus j laborers

g_{ik} = collection rate in pounds per hour for neighborhood type k for i days since last collection

The reasons that the waste generation is shown to be different for different types of collection frequency are pointed out in statistical studies by Quon, Tanaka and Charnes (1968). They show a significant increase in waste per household per week when collection frequency increases from once to twice a week.

Total collection cost in the subarea is C_i.

$$C_i = P_i k_i \left(4\frac{l_3}{g_{4,j}} + 3\frac{l_2}{g_{3,j}}\right)$$

dollars per week for a collection frequency of twice a week, and

$$C_i = p_i k_i' \left(3\frac{l_2}{g_{3,j}} + 2\frac{l_2}{g_{2,j}} + 2\frac{l_2}{g_{2,j}}\right)$$

dollars per week for a collection frequency of three times per week

The collection cost per ton = $\frac{C_i}{7p_i k_i} \times 2000$.

No extra charges were made for overhead or supervision as there was no way to obtain a reliable estimate of these figures.

4. Cost of Transport by Collection Vehicle from the Subarea to Transfer Facilities or Disposal Points

a) The speed in traffic of a collection vehicle is shown not to differ significantly between a loaded and an unloaded condition by Truitt (1968), whose histograms also indicate an average speed of 16 miles per hour.

b) The cost of the vehicle and crew per hour is the same as calculated in 3 (c) above for collection within a subarea.

c) The distance between the subarea and a disposal or transfer site is calculated in the following manner. A set of x, y coordinates are given for all transfer stations and disposal points. The location of the subarea is taken as its centroid. Since vehicles usually cannot travel in straight lines between points, travel distance is estimated using a metropolitan or rectangular measure in which the travel distance between a and b is

$$|X_a - X_b| + |Y_a - Y_b|$$

where $|\quad|$ means absolute value.

d) The average weight carried per 20 cubic yard collection vehicle at the disposal point is indicated in a histogram on page 161 in Truitt (1968). This is taken as 9,000 pounds per vehicle. Thus the cost per truck for travel from the disposal point to the collection area and return is measured in the following manner:

Define d = travel distance in miles in one direction

l_i = cost of vehicle + driver + i laborers per hour

s = travel speed in miles per hour

Then cost per truck load for the two way trip is

$$\frac{2\,d}{s} \times l_i$$

and since each truck carries on the average 9,000 pounds or 4.5 tons, the cost per ton is

$$\frac{1}{2.25} \times \frac{d}{s} \times l_i$$

Table 5-6. X, Y Coordinates for Proposed Transfer Stations*

Identification Symbol	Type	X Coordinate (feet)	Y Coordinate (feet)
RR	Transfer to railroad cars	24,000	21,000
A	Transfer to trailer trucks	16,000	11,000
B	Transfer to trailer trucks	14,000	22,000
C	Transfer to trailer trucks	28,000	20,500
D	Transfer to trailer trucks	22,500	7,000
E	Transfer to trailer trucks	37,000	9,500
F	Transfer to trailer trucks	34,500	21,000

*See map, Figure 5-3.

5. Transfer Facility Costs

Truitt (1968) listed four potential transfer sites which were located within the northwest quadrant, which will be used in this study as well. In addition, three more sites were chosen within the quadrant at various locations including one that is intended as a transfer to rail-haul selection. The coordinates X, Y, for each site are given in Table 5-6 and are shown on the map in Figure 5-3. The original Truitt sites are marked A, B, C, and D. The proposed additional vehicle transfer sites are E and F, and the railroad site is RR. The cost of building transfer facilities was also estimated by Truitt (1968) in the following manner:

Land Cost: Fixed cost of $40,000 below 200 tons per day capacity and a variable cost of $200 per ton of additional capacity above 200 tons per day.

Labor Costs: Three men at $20 per day for a transfer station of less than 200 tons per day or $60 per day. Four men at $20 per day or $80 per day for a greater capacity.

Utility Cost: $1200 per year.

Capital Cost of Structures: $125,000 for a capacity of less than 100 tons per day. Above 100 tons per day there is a variable cost of $500 per ton of additional capacity added to the fixed charge.

The total waste load per week from the northwest quadrant is estimated as 14 pounds per week for each of the 203,500 persons, or 1428 tons per week. The peak day load would occur in the early part of the week with three times a week collection. In this case on Monday and again on Tuesday, one half of the northwest quadrant is visited and 3/7 of the weekly waste load is collected. Thus the maximum daily load is

$$1/2 \times 3/7 \times 1425 = 306 \text{ tons per day}$$

The cost of a transfer facility as a function of weekly capacity is shown in Table 5-7. The yearly cost of the structure has been produced through discounting, assuming 30 year life and an interest rate of 10 percent. The yearly cost of the land is based on interest on investment and it is assumed that the land does not depreciate. No estimates were made on taxes lost on the land, since Truitt's sites A, B, C and D were chosen on land that was already public. It is assumed for simplicity that the same type of land is available at E and F.

6. Transfer from Transfer Station to Final Disposal Via Transport Vehicles and Disposal Costs

a) Transfer costs - The vehicles suggested for transport to the final disposal areas from the transfer station are of two types. One type is a 75 cubic yard tractor trailer with a weight capacity of an estimated 35,000 pounds. Truitt (1968) gives the cost of this

Table 5-7. Cost of Transfer Stations as a Function of Weekly Capacity

Weekly Capacity (tons/wk)	Total Structure Cost (dollars)	Total Land Cost (dollars)	Yearly Structure Cost* (dollars/yr)	Yearly Land Costs** (dollars/yr)	Utilities (dollars/yr)	Labor*** (dollars/yr)	Total Cost per Yr	Total Cost per Wk
600	125,000	40,000	13,300	4,000	1,200	18,720	37,220	719
900	150,000	40,000	16,000	4,000	1,200	18,720	39,920	770
1,200	175,000	40,000	18,700	4,000	1,200	18,720	42,620	820
1,500	200,000	50,000	21,400	5,000	1,200	24,960	52,560	1,010
1,800	225,000	60,000	24,100	6,000	1,200	24,960	56,260	1,080
2,100	250,000	70,000	26,800	7,000	1,200	24,960	59,960	1,145

*Yearly structure cost based on capital recovery factor - 10 percent - 30 years.

**Yearly land cost based on 10 percent interest rate for opportunities foregone.

***Based on 6 days a week, 52 weeks a year operation.

vehicle at \$11.00 per hour. Using \$3.33 per hour for a driver gives a cost of \$14.33 per hour of truck use. The average speed is estimated at 16 miles per hour. The other type would be rail-haul from the transfer station to some far distant point for disposal. Philadelphia, Pennsylvania, as reported by PUBLIC WORKS magazine, (Anon, 1968), presently does this at a contracted rate of \$5.40 per ton for shipping and disposal in abandoned coal mines 200 miles away in western Pennsylvania. This cost will be used as an estimate of rail-haul transport and disposal costs.

b) Estimates of tractor trailer vehicle cost per ton.

Define X_F, Y_F = coordinates of the facility
X_D, Y_D = coordinates of the disposal point
c = capacity of the vehicle in pounds
s = travel speed in miles per hour
d = distance to be traveled - one way
l = cost per hour of vehicle plus labor

then cost per ton = $(2 \frac{d}{s} \times \frac{l}{c} \times 2000)$

where d is measured as before by a rectangular metric

$$d = |X_F - X_D| + |Y_F - Y_D|$$

where $|\quad|$ means absolute value of the difference.

c) Costs and locations of disposal areas - The location of the disposal points are given as the two existing incinerators, the Reedbird and the Pulaski. One additional alternative, that of rail-haul, is also included. The unit cost used as based on the previously mentioned data accounts for a rail-haul of approximately 200 miles. Therefore, it is not necessary to locate the rail-haul disposal facility. The x-y coordinates for the two existing facilities and the unit cost of operation as given in the 1965 Public Works Report of the City of Baltimore are shown in Table 5-8. Refer to Figure 5-3 for location in relation to the collection area.

Table 5-8. Characteristics of Baltimore Incinerators

Name	Location		Capacity (tons/day)	Cost per Ton*
Reedbird - #3	x = 26,000	y = 45,000	600	3.60
Pulaski - #4	x = 49,000	y = 26,000	800	2.80

*From Anon. (1966a), Baltimore Public Works Department Annual Report

D. Results of the Analysis

1. Explanation of Symbols

A total of 60[1] runs were made with the facilities location model and the data case of section C. These runs represent a broad spectrum of changes in parameters and conditions in order to show the sensitivity of the system and to estimate the order of magnitude of the cost of proposed system policy and operational changes. These runs are summarized in Table 5-9. The headings of each column are explained and referenced in the following manner:

Type of Transfer Station - For most runs, seven transfer station site alternatives were allowed, A, B, C, D, E, F and RR. These symbols and the locations of the sites have been shown previously in Table 5-6. It was assumed in most runs that the same capacity facility was proposed at each site. Any deviation from this policy, or in the number of sites available, is noted in the REMARKS column beside the run.

Collection Frequency - Two cases were considered: twice a week (2) and three times a week (3).

Collection Costs - These are the costs of picking up the solid waste within the collection areas. The basis for estimating this cost is shown in section C (3) of this chapter. Units are in dollars per week.

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[1] Each run took an average of 45 seconds, or for the 60 runs a total of 45 minutes of execution time on the IBM 7094.

Transfer Cost from Collection - This represents the transportation costs incurred by the collection vehicles in hauling their collected load from the area where it was picked up to the point where it will be discharged, and the return trip to the collection area. The point of discharge in most cases may be either a transfer facility or a final disposal point. In most cases when transfer stations were built not all collection vehicles used them, for it was still economically feasible for some collection subareas to ship directly to final disposal. Units are dollars per week.

Facility Costs - Cost per week of building and operating a facility of a given capacity. Computations to arrive at these figures are shown in Table 5-7.

Transfer from Transfer Stations - This category represents the cost incurred using specialized tractor trailer transport vehicles to move wastes from the transfer site to the final disposal site. If rail-haul is used, such transfer costs do not appear in this category but as part of the disposal cost, since there is no way to break apart the unit cost figure for rail-haul and disposal. Units are dollars per week.

Disposal Costs - This reflects the unit cost of disposal. In all cases where local disposal was chosen, the Pulaski incinerator was chosen over the Reedbird incinerator because of its preferable location with regard to the northwest quadrant that makes up the study area, and because of the lower unit costs of disposal at the newer Pulaski incinerator. In the case where rail-haul was chosen

Table 5-9. Description of Runs with Baltimore Data

Run No.	Type of Transfer Station Proposed Tons/Wk	Collection Freq. Times per Wk	Collection Cost Dollars per Wk	Transfer from Collection Dollars per Wk	Facility Costs Dollars per Wk	Transfer from Transfer Station Dollars per Wk	Disposal Cost Dollars per Wk
1	1200	2	8,103	1936	820	742	3998
2	NONE	3	9,361	4040	0	0	3998
3	600	3	9,361	2226	719	540	3998
4	900	3	9,361	1847	770	630	3998
5	900	2	8,103	2097	770	630	3998
6	600	2	8,103	1490	1438	840	3998
7	1200	3	9,361	1699	820	742	3998
8	1500	3	9,361	1620	1010	765	3998
9	1500	3	9,361	1780	1010	914	3998
10	NONE	3	9,361	5060	0	0	3998
11	NONE	3	9,361	6087	0	0	3998
12	1500	3	9,361	1780	1010	1200	3998
13	1500	3	9,361	1780	1010	1600	3998
14	NONE	3	9,361	1773	1010	0	7711
15	NONE	3	9,361	1773	1010	0	7711
16	1500	3	9,361	1780	1010	7000	3998
17	NONE	2	8,103	4563	0	0	3998

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Fac'lty Built	Load Sent to Fac'lt's Tons/Wk	Load Directly to Disposal Tons/Wk	Total Load Tons/Wk	Total Cost Dollars/Wk	Remarks
B	1060	368	1428	15,599	
NONE	0	1428	1428	17,399	
A	600	828	1428	16,844	
B	900	528	1428	16,606	
B	900	528	1428	15,598	
A, C	600 600	228	1428	15,869	
B	1060	368	1428	16,620	
B	1093	335	1428	16,754	
C	1428	0	1428	17,063	Av'ge haul distance 10 miles
NONE	0	1428	1428	18,419	Av'ge haul distance 10 miles
NONE	0	1428	1428	19,449	Av'ge haul distance 12 miles
C	1428	0	1428	17,349	Av'ge haul distance 12 miles
C	1428	0	1428	17,749	Av'ge haul distance 14 miles
RAIL HAUL	1428	0	1428	19,855	Av'ge haul distance 14 miles
RAIL HAUL	1428	0	1428	19,855	Av'ge haul distance 16 miles
C	1428	0	1428	18,149	Av'ge haul distance 16 miles
NONE	0	1428	1428	16,664	

Table 5-9. Description of Runs with Baltimore Data (continued)

Run No.	Type of Transfer Station Proposed Tons/Wk	Collection Freq. Times per Wk	Collection Cost Dollars per Wk	Transfer from Collection Dollars per Wk	Fac'lty Costs Dollars per Wk	Transfer from Transfer Station Dollars per Wk	Disposal Cost Dollars per Wk
18	1500	2	8,103	1846	1010	765	3998
19	1500	2	8,103	2020	1010	900	3998
20	NONE	2	8,103	5714	0	0	3998
21	NONE	2	8,103	1967	1010	0	7711
22	1500	2	8,103	2020	1010	1350	3998
23	1500	2	8,103	2020	1010	1800	3998
24	NONE	2	8,103	1967	1010	0	7711
25	NONE	2	8,103	1967	1010	0	7711
26	1500	2	8,103	2020	1010	2250	3998
27	1800	2	9,245	2028	1080	841	4398
28	1800	2	10,107	2220	1080	920	4807
29	1800	2	10,967	2408	1080	999	5216
30	1800	2	11,795	2590	1080	1073	5611
31	1800	2	12,674	2781	1080	1154	6028

Fac'l'ty Built	Load Sent to Fac'l't's Tons/Wk	Load Directly to Disposal Tons/Wk	Total Load Tons/Wk	Total Cost Dollars/Wk	Remarks
B	1093	335	1428	15,722	
C	1428	0	1428	16,031	Av'ge haul distance 10 miles
NONE	0	1428	1428	17,815	Av'ge haul distance 10 miles
RAIL HAUL	1428	0	1428	18,791	Av'ge haul distance 12 miles
C	1428	0	1428	16,481	Av'ge haul distance 12 miles
C	1428	0	1428	16,931	Av'ge haul distance 14 miles
RAIL HAUL	1428	0	1428	18,791	Av'ge haul distance 14 miles
RAIL HAUL	1428	0	1428	18,791	Av'ge haul distance 16 miles
B	1428	0	1428	17,381	Av'ge haul distance 16 miles
B	1202	369	1571	17,592	Increase waste load by 10%
B	1315	402	1717	19,134	Increase waste load by 20%
B	1427	436	1863	20,670	Increase waste load by 30%
B	1534	470	2004	22,149	Increase waste load by 40%
B	1649	504	2153	23,717	Increase waste load by 50%

Table 5-9. Description of Runs with Baltimore Data (continued)

Run No.	Type of Transfer Station Proposed Tons/Wk	Collection Freq. Times per Wk	Collection Cost Dollars per Wk	Transfer from Collection Dollars per Wk	Facility Costs Dollars per Wk	Transfer from Transfer Station Dollars per Wk	Disposal Cost Dollars per Wk
32	1800	2	13,501	2965	1080	1229	6428
33	1800	2	14,356	3212	1080	1260	6829
34	1800	2	15,220	3507	1080	1260	7240
35	1800	2	16,074	3818	1080	1260	7646
36	1800	2	8,103	1700	1080	778	3998
37	1800	2	8,103	1688	1080	454	3998
38	1800	2	8,103	1684	1080	344	3998
39	1800	2	8,915	1846	1080	765	3998
40	1800	2	7,310	1846	1080	765	3998
41	1800	2	6,660	1846	1080	765	3998
42	1500	2	11,264	2918	1010	1050	6028
43	2100	2	15,043	3808	1145	1470	8050

Fac'l'ty Built	Load Sent to Fac'l't's Tons/Wk	Load Directly to Disposal Tons/Wk	Total Load Tons/Wk	Total Cost Dollars/Wk	Remarks
B	1756	538	2294	25,198	Increase waste load by 60%
B	1800	639	2439	26,737	Increase waste load by 70%
B	1800	706	2586	27,869	Increase waste load by 80%
B	1800	931	2731	29,018	Increase waste load by 90%
A	1112	316	1428	15,659	Transfer vehicle speed 20 mph
A	1135	293	1428	15,323	Transfer vehicle speed 30 mph
A	1147	281	1428	15,209	Transfer vehicle speed 40 mph
B	1093	335	1428	16,604	Vary collection rate by -10%
B	1093	335	1428	14,999	Vary collection rate by +10%
B	1093	335	1428	14,349	Vary collection rate by +20%
B	1500	653	2153	22,270	B is only alternative Increase waste load by 50%
B	2100	775	2875	24,516	B is only alternative Increase waste load by 100%

Table 5-9. Description of Runs with Baltimore Data (continued)

Run No.	Type of Transfer Station Proposed Tons/Wk	Collection Freq. Times per Wk	Collection Cost Dollars per Wk	Transfer from Collection Dollars per Wk	Facility Costs Dollars per Wk	Transfer from Transfer Station Dollars per Wk	Disposal Cost Dollars per Wk
44	1500	3	9,361	1620	1010	765	3998
45	1500	2	8,103	1498	2020	851	3998
46	1500	3	9,361	1230	2020	591	3998
47	1500	2	8,103	1754	1010	940	3998
48	1500	3	9,361	1780	1010	714	3998
49	1500	2	8,103	1846	1265	765	3998
50	1500	3	9,361	1620	1265	765	3998
51	1500	2	8,103	1846	1518	765	3998
52	1500	3	9,361	1620	1518	765	3998
53	1500	2	8,103	4564	0	0	3998
54	1500	3	9,361	4040	0	0	3998
55	1500	3	9,828	1700	1010	804	4197
56	1500	3	10,291	1780	1010	841	4398

Fac'l'ty Built	Load Sent to Fac'l't's Tons/Wk	Load Directly to Disposal Tons/Wk	Total Load Tons/Wk	Total Cost Dollars/Wk	Remarks
B	1093	335	1428	16,752	B is only alternative Increase waste load 0%
B, D	746 411	271	1428	16,471	Political jurisdiction case
C, F	700 415	313	1428	17,200	Political jurisdiction case
A	1044	384	1428	15,805	Deny B as an alternative
C	1428	0	1428	16,863	Deny B as an alternative
B	1093	335	1428	15,977	Increase fixed costs by 25%
B	1093	335	1428	17,009	Increase fixed costs by 25%
B	1093	335	1428	16,230	Increase fixed costs by 50%
B	1093	335	1428	17,262	Increase fixed costs by 50%
NONE	0	1428	1428	16,664	Increase fixed costs by 100%
NONE	0	1428	1428	17,399	Increase fixed costs by 100%
B	1148	351	1499	17,539	Increase waste load by 5%
B	1202	369	1571	18,321	Increase waste load by 10%

Table 5-9. Description of Runs with Baltimore Data (continued)

Run No.	Type of Transfer Station Proposed Tons/Wk	Collection Freq. Times per Wk	Collection Cost Dollars per Wk	Transfer from Collection Dollars per Wk	Facility Costs Dollars per Wk	Transfer from Transfer Station Dollars per Wk	Disposal Cost Dollars per Wk
57	1500	3	10,758	1861	1010	880	4594
58	NONE	3	9,828	4245	---	---	4197
59	NONE	3	10,290	4443	---	---	4398
60	NONE	3	10,757	4646	---	---	4594

Fac'l'ty Built	Load Sent to Fac'l't's Tons/Wk	Load Directly to Disposal Tons/Wk	Total Load Tons/Wk	Total Cost Dollars/Wk	Remarks
B	1257	384	1641	19,103	Increase waste load by 15%
NONE	0	1499	1499	18,270	Increase waste load by 5%
NONE	0	1571	1571	19,132	Increase waste load by 10%
NONE	0	1641	1641	19,998	Increase waste load by 15%

as the disposal method, the disposal costs include the cost of hauling to the distant disposal point. Where local disposal was chosen, the cost includes only the actual disposal process. The sources of these costs are shown in section C (6) of this chapter and are in dollars per week.

Facilities Built - If transfer sites were chosen by the models, this indicates at which sites transfer stations of the given capacity would be built.

Load Sent to Facilities - Tons of waste per week from the collection areas that would be sent to transfer facilities rather than directly to final disposal.

Load Directly to Disposal - Tons per week sent directly to disposal without trans-shipment to a transfer site.

Total Load - The total waste load in tons per week collected. Represents the sum of the previous two categories.

Total Cost - The sum of the collection, transfer from collection, facility, transfer from facilities and disposal costs. Units are dollars per week.

2. Explanation of Runs

Estimation of Present System Costs and Verification - The most important aspect of the study is to establish some bench mark against which the results of changing the system may be measured to determine their efficiency. The present system runs with two times a week collection frequency so the computation of this cost will serve not only as a bench mark but also as a check on the validity of the model, since published figures are available for the cost of running the actual system (Anon, 1966a). Run number 17 represents this verification since its data inputs are the best estimates of all data, collection frequency is twice a week and no transfer stations are allowed. The yearly cost of collection by the city from the northwest division for the year 1965 was $722,000. The model reports this cost as $12,666 per week, or $688,632 per year, which is a difference of 4.5 percent. The difference between the two estimates is slight and may be explained by the fact that 1960 population estimates were used in the model while the actual cost figure represents the cost of collecting from the 1965 population of the same region. Since the northwest division of the city, particularly at its outermost fringes, is gaining in population, it would be expected that the model estimates would be slightly lower.

Estimation of a Three Times a Week Collection Frequency Bench Mark - In run number 2, an estimate was made of the cost of three times a week collection frequency without transfer stations, with a resulting total system cost of $17,399 per week. When compared to the twice a week bench mark of $16,664 per week this shows only a 4.6 percent increase in cost to increase in collection frequency. However, this estimate is really only a lower bound on the actual cost of three times a week collection and thus should be viewed with care. There are several assumptions buried in the calculation of the cost of three times a week collection that should be clearly stated. The first is that the routes for the vehicles will be changed from those of the twice a week collection vehicles, to insure that the truck is close to capacity before it makes the trip to the transfer or disposal point. As noted, there is no instrumentation on a vehicle which tells the driver the present load of his vehicle. Routes are usually assigned in terms of definite areas, with the driver returning when he reaches a certain place. Since, in three times a week collection, less waste is generated in a given area as there has been less time since the last collection, the vehicle must travel farther to get a complete load. The load on a vehicle is an important factor in the calculation. The average load for twice a week collection was 9000 pounds. Should the average load for three times a week collection drop because routes are improperly designed, say by 10 percent, the cost difference between twice and three times a week would rise to 8 percent. Should the average collected weight drop by 25 percent, the cost difference between twice and three times a week collection would rise to 14 percent. Thus extreme care must be taken in designing the routes to make sure that enough load is picked up before transportation. A factor to be watched in the design of the routes is the time factor. Most operations work under a "no overtime" constraint, which means that at a certain time the truck will return, regardless of its present load. Since it takes longer to collect under a three times a week policy careful attention must be paid to the time length of the routes. Models as suggested in Chapter IV on routing would be used to assign routes correctly.

A second assumption that has been made in these computations is that the waste load is the same for twice a week and three times a week collection. This may not be a valid assumption, but no published evidence exists to disprove it. However, work by Quon et al. (1968) based on actual observations in a controlled experiment in Chicago shows significant increases in waste loads when collection frequency was increased from once to twice a week. They found waste load increases in the order of 30 to 50 percent. To test the effect of waste load increase on the three times a week collection calculation, runs 58, 59 and 60 were made. These runs represent three times a week collection with no transfer, and a waste load increase of 5, 10 and 15 percent respectively. These runs show that the estimation of the difference between twice and three times a week collection is quite sensitive to the assumption made about waste load. These computations are shown in Table 5-10. Great care should be taken in estimating the difference between twice and three times a week collection, as the cost difference is very sensitive to the new waste load estimates.

However, in measuring and comparing differences between individual runs when both have the same collection frequency, the type of assumption made does not have a serious effect on the calculation since the same assumptions appear in both. The remainder of the runs made with a collection frequency of three times a week are all made with the assumption that the average truck load does not decrease from twice a week collection and that individual waste load generated does not change.

Table 5-10. Different Waste Load Assumptions and Their Effect on the Percent Difference Between Twice and Three Times a Week Collection with no Transfer

Waste Load Assumption	Total System Cost Dollars per Week	Cost Difference Between Twice a Week and Three Times a Week Collection
Same as 2 times a week	17,399	4.5%
5% greater	18,270	10.0%
10% greater	19,132	15.0%
15% greater	19,998	20.0%

Is Transfer Feasible? - With the best data available for the various cost parameters and for the cost of facilities, a series of runs was carried out with different capacity transfer stations at the proposed transfer sites to show if transfer stations lower the cost of collection. In every case they did. The results of the analysis also give some interesting insights into the system. Runs 3, 4, 7 and 8 were made for three times a week collection and for 4 different sized transfer stations ranging from 40 percent of the estimates of weekly waste load (600 tons per week) to over 100 percent (1500 tons per week). Similarly, runs 6, 5, 1 and 18 were made for the twice a week collection rate and the results of all these runs are presented in Figure 5-4. First, notice that beyond the 600 ton per week alternative, the cost function for the higher capacity stations is more favorable and quite flat, indicating little sensitivity to size within that range. More important, for each solution only one transfer site was chosen and for all six cases that site was B. Thus this preliminary analysis would seem to favor B for the present system. In all cases, the building of only one transfer station indicated that some waste load was still going directly to the final disposal point without transfer. Both the 1200 and 1500 ton capacity per week facilities were used at less than capacity with about 40 percent of the waste in both cases going directly to disposal. Later runs will show that this proportion is very sensitive to the location of the final disposal points. The cost for three times a week collection with transfer is comparable with the present two times per week collection without transfer. Later calculations will show how comparable these costs are if waste load should be increased for the higher frequency case.

For the smaller capacity (600 tons per week) stations, a different alternative (A) was chosen and in one case two alternatives (A and C) were chosen. In general, the difference between the solution with transfer stations and the one without is in the range of

4 to 7 percent with the two times a week collection frequency allowing a better decrease. The absolute difference between the transfer and non-transfer solution is a rough measure of how much facility costs can increase before transfer is no longer favorable. More will be said about this later.

Effect of Increased Haul Distance to Final Disposal - The present average haul distance from the northwestern division to disposal at the Pulaski incinerator was shown to be 8 miles one way by Truitt (1968) and was verified by this author. Now suppose that the haul distance was increased. Such a case is possible because as the city expands, the tendency is to move waste disposal farther out in order to avoid creating a nuisance to nearby neighbors, and because of the lack of availability of unused sites near the city. Runs were made to see what the effect would be if the disposal points were moved so that average haul distance was increased to 10, 12, 14 and 16 miles. In this analysis there is an additional possibility which was not considered in Truitt's work. This is rail-haul to far distant disposal points. As average haul distance increases, transfer facilities become more and more favorable until finally the most dominant choice is rail-haul. A set of runs was made to see how moving the disposal point would affect a system with and without transfer stations. Runs 10, 11, 14 and 15 represent the case of increasing haul distance without allowing transfer, for three times a week collection frequency. Runs 9, 12, 13 and 16 allow transfer under the same conditions. Figure 5-5 shows the results of these runs compared to the cost of the rail-haul alternative. It is evident that as the haul distance increases, transfer becomes more favorable to non-transfer. In fact with non-transfer, rail-haul becomes feasible at 12.9 miles average haul distance. If transfer is allowed, rail-haul is not a suitable alternative until almost 26 miles average haul distance. This indicates that as pressure mounts to move disposal out farther from the city, a transfer facility system would preclude rail-haul for some time, given that the relative costs of the alternatives would remain the same. Further, the transfer site chosen in each of the runs was the same site C, indicating that the site selection in this case is quite stable. Also, as soon as the disposal point has been moved only 2 miles, all waste in the system goes through the transfer facilities and none is taken directly to the disposal point. This is the reason C was chosen for this case while B was chosen for the previous cases with present location of facilities. The choice between B and C favors B only slightly, and when conditions are changed to cause more load to go through a facility, C becomes the best alternative.

In Figure 5-6, results of runs 19, 20, 21, 22, 23, 24, 25 and 26 are plotted to show how a twice a week collection system would react under the same haul distance changes. The results are quite similar, with C being chosen as the transfer site and the rail-haul alternative becoming feasible at a slightly smaller haul distance than in the previous case.

Thus it would appear that the questions concerning transfer and rail-haul are intimately linked. Under present conditions, rail-haul is not a feasible alternative. At a cost of $18,791 per week for the present system it represents a 12.8 percent increase in cost over the system with no transfer, and an 18.3 percent increase over the system with transfer. However, if haul distance should increase, rail-haul would become a feasible alternative in a fairly short time if no transfer is established. If transfer facilities are built they reduce costs to the extent that rail-haul would not be feasible for a considerable time. As for transfer itself, it does exhibit some, but not extraordinary, savings over non-transfer for the present system. For future conditions, however, these savings will increase, thus making transfer even more favorable.

Sensitivity to other Parameters - Other runs were made to show how sensitive the system was to estimates of the various parameters. Since waste load estimation in three times a week collection has been shown to have an effect on the solution, several additional runs were made to study this situation. Runs 55, 56 and 57 represent runs with transfer facilities allowed, three times a week collection frequency and an increase in the waste load of 5, 10 and 15 percent, while runs 58, 59 and 60 represent the system without transfer. The results are shown in Table 5-11. The cost of the transfer station decision is even more sensitive than the

Table 5-11. Different Waste Load Assumptions and Their Effect on Percent Difference Between Twice and Three Times a Week Collection with Transfer

Increase in Waste Load (%)	Increase in Cost with no Transfer (%)	Increase in Cost with Transfer (%)	Site Alternative Selected
0	4.5	6.4	B
5	10.0	11.4	B
10	15.0	16.6	B
15	20.0	21.5	B

non-transfer solution. However, in all cases the site selected was B, indicating that only the cost estimate but not the site chosen is sensitive to this parameter. Twice a week collection was tested for its sensitivity to waste load increase in runs 27, 28, 29, 30, 31, 32, 33, 34 and 35, where the waste load was progressively increased until it almost doubled. Results of these runs are shown in Figure 5-7. Even though waste load was increasing, the solution cost responded linearly and the same alternative B, was chosen for all solutions. Even when waste load was almost double the transfer station capacity it was not optimal to build a second facility at a different site.

Runs 36, 37 and 38 show how sensitive the solution is to the speed of the transfer vehicle from the transfer facility to the disposal point. Results are plotted in Figure 5-8. Present estimates place this speed at 16 miles per hour since this is the approximate speed in traffic of the collection vehicles, and the transfer vehicles are larger tractor trailers which must operate under largely the same traffic conditions. The plot indicates that an increase in the traffic speed from 16 to 30 miles per hour would bring about a decrease of about 7.5 percent in the cost of the solution. However, it would be expected that the cost of the remedial measures necessary to bring about this increase in speed would more than outweigh any savings in transportation costs.

An attempt was also made to see how collection rates affect the solution. In runs 39, 40 and 41 collection rates are varied by -10 percent, +10 percent and +20 percent. Increasing or decreasing the collection rate by 10 percent leads to about a 5 percent change in the solution cost, which falls into the range of being somewhat sensitive. Thus care should be taken in estimating these rates. Further, work rules that tend to increase collection rate without too great an increase in cost or inexpensive time saving devices would also bear some investigation.

The effect of changing the estimates of the facilities costs is also of interest. This gives some feeling for how carefully cost estimates have to be made for preliminary studies. Runs 49, 50, 51, 52, 53 and 54 show how fixed cost increases of 25, 50 and 100 percent would affect the solution. At about a 75 percent increase in fixed costs, transfer facilities are no longer feasible, but up to that point the same facility was chosen each time. A rough estimate of how much facilities costs could increase could have been found from earlier runs simply by looking at the absolute difference in cost between the transfer and non-transfer solutions. If the facility cost increase exceeded this difference, transfer was no longer feasible.

Political, Aesthetic and Regional Constraints - Many times constraints occur which preclude the use of an alternative that would otherwise seem to be the best from an economic point of view. Examples are political constraints when a site might better be put to use for other municipal functions, aesthetic constraints where the neighborhood around a site strongly opposes a facility there, and regional constraints which do not allow for cooperation between different political subdivisions. Consider first an example where a particular site is not allowed to be used. The results of the analysis to this point show that B is the best transfer site. Runs 47 and 48 show what happens when this site is excluded from the set of alternatives. In the case of two times a week collection, Site A is chosen and the increase in the cost is about $60 per week. For three times a week collection the site chosen is site C and the difference is about $110 per week. Both of these changes are minuscule compared to the total cost of operating the system. This indicates that there may be readily available alternatives to the best site that will not raise the cost of the solution significantly. Thus the cost of satisfying additional constraints which prohibit certain sites is not particularly high, and the decision maker has a great deal of leeway in choosing between alternatives when other criteria must also be considered.

The effect of regionalization will only be treated briefly in this example, as neither the time nor data were available for assembling an extremely large regional case. However, the question can be looked at implicitly by assuming that the northwestern division is a unified region, and asking how much costs would increase if it were subdivided into two non-cooperating regions. Suppose the region were divided into two so that the northern areas could use only transfer sites A, D and E, and the southern areas could use only sites B, C and F. The results of these runs are shown in Table 5-12. In this case subdividing the region still makes transfer feasible, but only at an extremely slight advantage over no transfer at all. This would suggest that regionalization would mean increased savings. Although this analysis was not extended to the entire city, it might be expected that each of the administrative divisions would not have a transfer facility, but only two or three would be built.

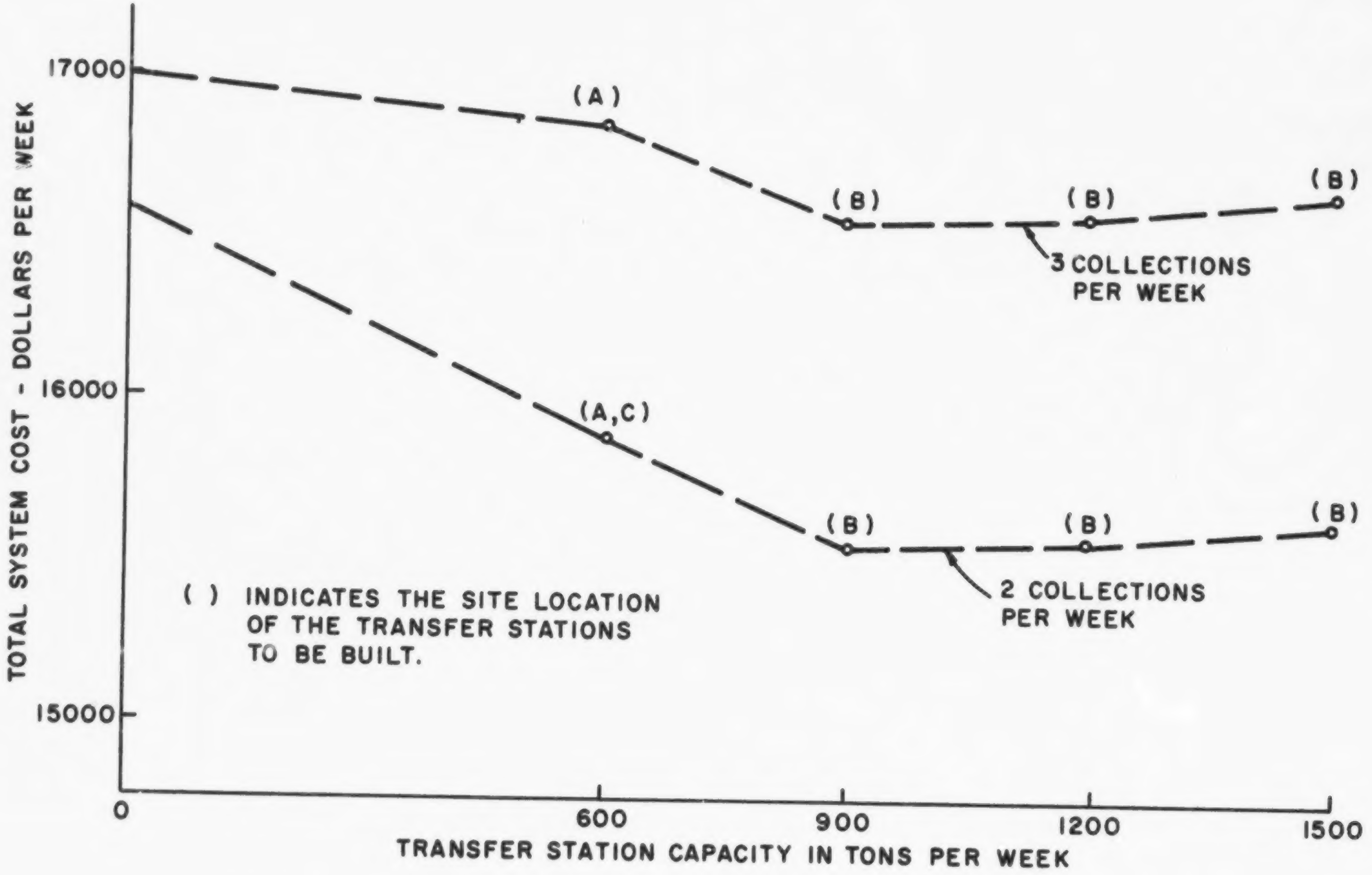

Figure 5-4. Plot of Total System Cost vs. Size of Transfer Station

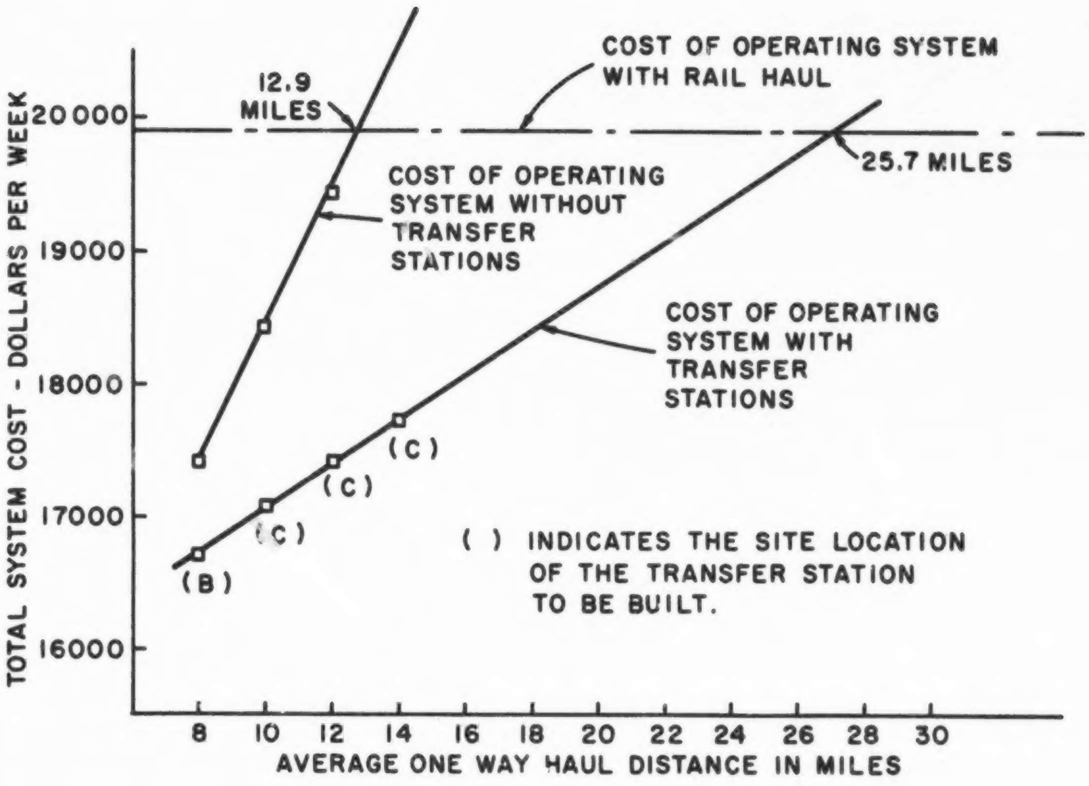

Figure 5-5. Plot of Average Haul Distance vs. Cost of Total System. Collection Frequency = Three Times per Week.

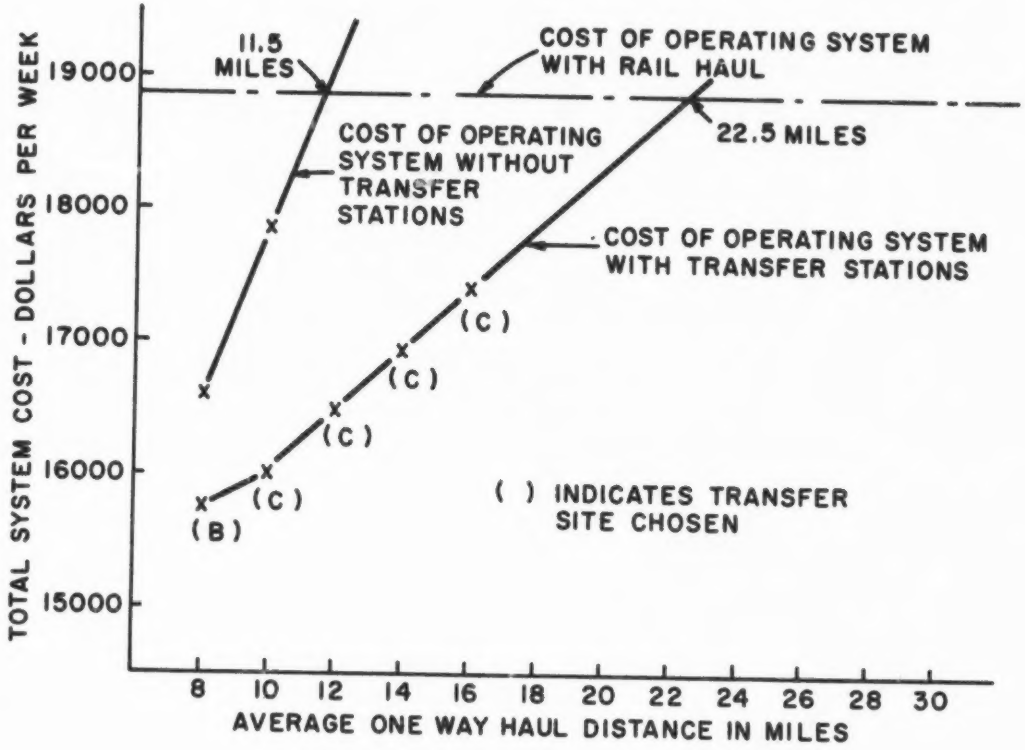

Figure 5-6. Plot of Average Haul Distance vs. Cost of Total System. Collection Frequency = Two Times per Week.

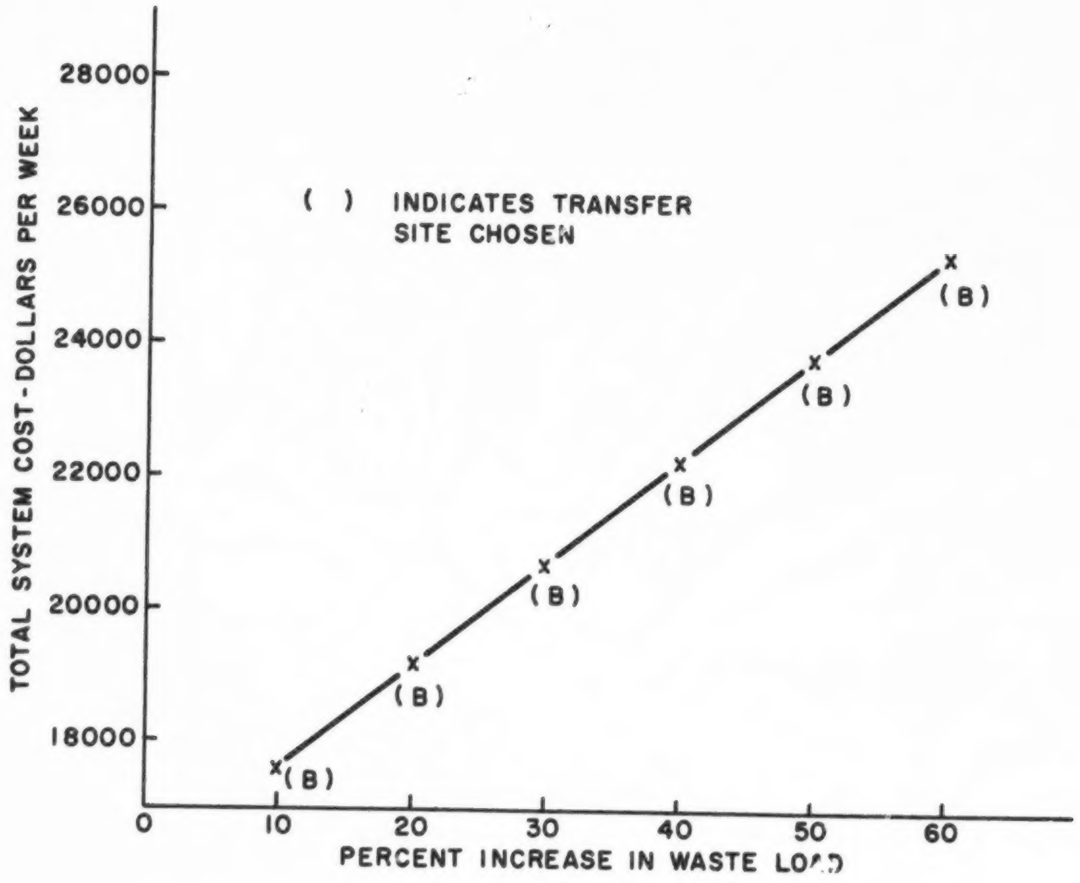

Figure 5-7. Plot of Cost of Solution vs. Estimate of Percent Increase in Waste Load.

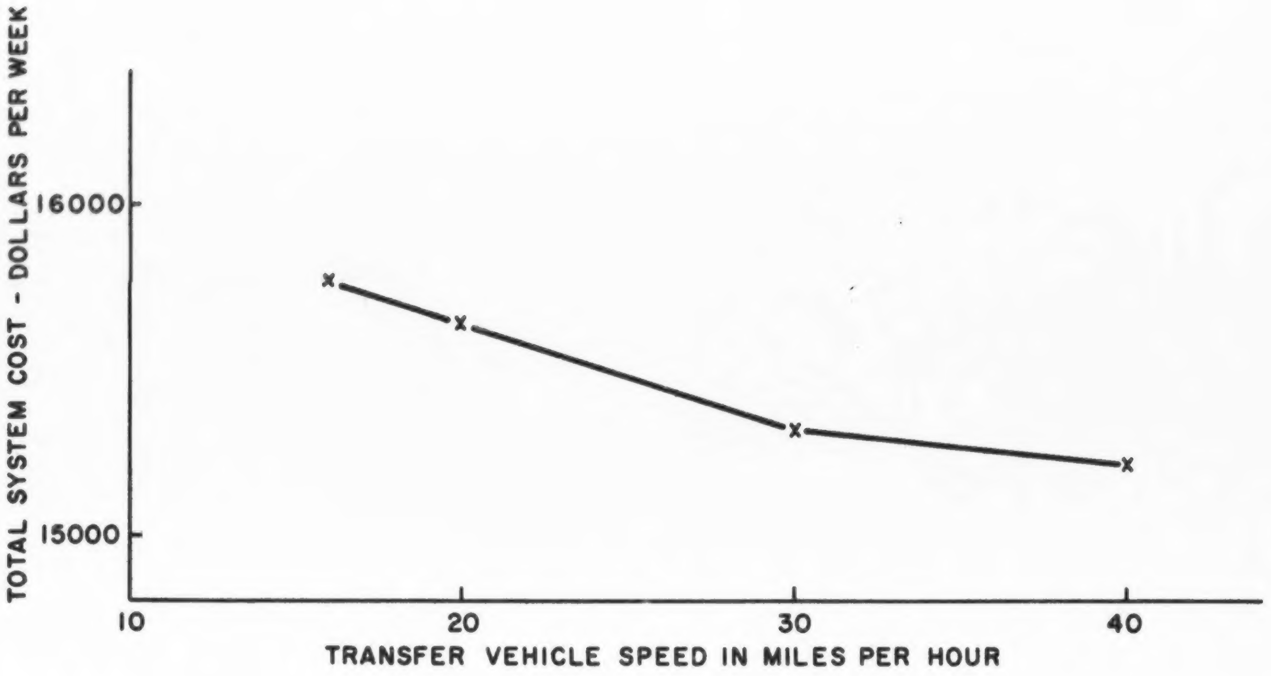

Figure 5-8. Plot of Transfer Speed in MPH vs. Cost of Solution

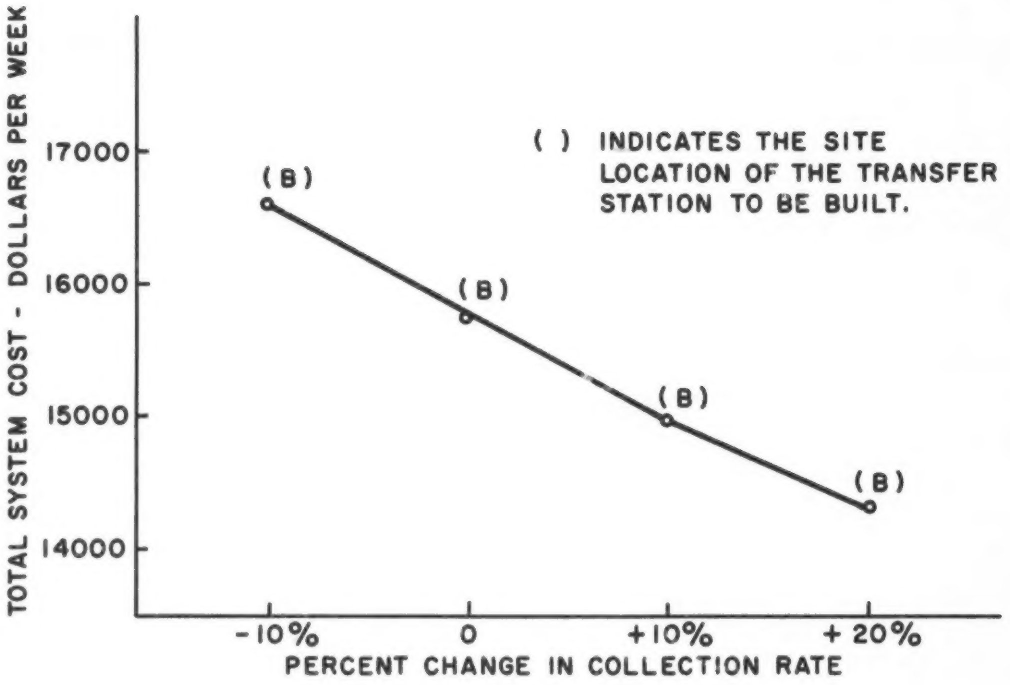

Figure 5-9. Plot of Varying Collection Rate vs. Cost of Solution.

Table 5-12. Effects of Regional Cooperation

Collection Frequency	Northwestern Division is Two Districts with Transfer		Northwestern Division is One District Without Transfer		Northwestern Division Without Transfer
	Cost/Week	Sites	Cost/Week	Site	Cost/Week
2	$16,471	B and D	$15,720	B	$16,664
3	17,200	C and F	16,752	B	17,399

E. Summary of the Experimental Analysis

The following general statements about the system can be made, based on the rough analysis with estimated data for the northwestern division. These runs have been intended as a demonstration of the techniques of this thesis, rather than a detailed analysis of the system. The actual findings would need verification with better cost data before they could be viewed as completely reliable.

1. The building of transfer facilities appears to be feasible and would result in an annual savings, under present conditions, of 7 percent in the total cost of operating the solid waste collection system. This saving would be expected to increase in the future, as the system expands and grows.

2. For the northwestern division only one transfer facility would be built, with site B chosen as the location. Site C is also a good alternative and selecting C over B would show little disadvantage.

3. The most favorable size of the facility to be built is 1500 tons per week capacity. At this size some of the waste material will be taken to transfer and some will still be taken directly to the disposal point. These assignments are shown by the model.

4. A rail-haul alternative would cost about 12 percent more to operate than the present system and about 18 percent more than the system with a transfer facility. Haul distance would have to increase an average of from 3.5 to 5 miles for rail-haul to be favorable without transfer, and from 14 to 18 miles to be favorable with transfer.

5. The cost of changing from two times a week frequency of collection to three times a week is difficult to estimate because of the sensitivity of this calculation to estimates of how much the waste load would be with three times a week collection. Estimates of the added cost range from 4.5 percent if there is no increase in the waste load, up to 21.5 percent with a 15 percent increase in the waste load. All of these figures are based on the assumptions that routes for vehicles under three times a week collection be redesigned for efficiency purposes. It is strongly advised that more studies of the waste load question in this region be carried out before any firm decision be made on changing collection frequency.

6. While differences in assumptions make estimating the cost of some programs difficult, the site selection B, for a transfer facility is remarkably insensitive to parameter change. Regardless of the collection frequency, site B is the best site for the transfer facility.

7. The system shows some sensitivity to collection rates, which indicates that care should be taken in measuring them and investigation of means of altering them will be of some help.

8. If for some reason site B could not be used, site C first and then site A are good alternatives, and it would cost little extra to change the selection to them. The effects of regionalization are such that the northwestern division is better operated as one division than as two. It is further expected that extension of this type of analysis to the entire city might show even greater efficiency with transfer, and this is recommended as the next step in the analysis procedure.

9. The amount of computer time necessary for this analysis of 60 runs was 45 minutes, using the IBM 7094. At commercial rates of $500 per hour, this represents a cost of approximately $375.

CHAPTER VI. CONCLUSIONS AND EXTENSIONS

The goal of this thesis has been the development of tools to gain a better understanding of a large-scale public system. Thus it is on the basis of the results of the example rough analysis of the Baltimore, Maryland, solid waste collection system in Chapter V that the success of such an undertaking should be measured. While most of the data used for the analysis were less than elegantly obtained, a firm body of facts and conclusions emerge from the exercise. Answers to many of the basic questions with which the analyst is concerned were found. Most important, a feeling for the sensitivity of the system to many of its parameters was established. The decision to change from twice a week to three times a week collection was shown to be quite sensitive to estimates of waste generation under the two regimes. Thus, it is recommended that additional study in this area be carried out before a decision is made about a policy change. On the whole, however, the problem of choosing transfer facility location appears to be quite stable, with the same location being elected under a severe change in system conditions.

Some drawbacks in the analysis were encountered because of the need to subdivide the problem in order to solve it. At one point it was necessary in the study of facility location to assume that the new routes for vehicles would be set up efficiently if service policy whould change. However, the model could not determine how this was to be done, and the problem would have to be looked at as a separate vehicle scheduling problem as described in Chapter IV.

From a computational point of view the analysis in Chapter V was a success. Sixty different problems, each involving generating data for forty waste sources and considering nine intermediate alternatives, were solved in about three-quarters of an hour on the IBM 7094. This represents a cost at commercial rates of less than 400 dollars, or about two weeks' salary for a medium level engineering analyst. In terms of the information generated, it seems like a considerable bargain.

Looking at the individual models developed, the large-scale "flow of goods" models developed for the facility location problem of Chapter II and the multi-commodity truck assignment problem of Chapter III proved to be the most successful. When programmed for the computer, they were of sufficient size and speed to handle a very large complex system and find solutions quickly. Since much of the value of the models is in repeated solution with changing parameters to determine sensitivity, efficient solution time is of great importance. The vehicle scheduling models of Chapter IV do not display the same qualities. People working in this field have uniformly found that securing an optimal solution to even a small-scale problem in a reasonable amount of time was difficult to impossible. Although the author's work in a problem area of this general category extended theoretical concepts involving the use of more than one terminal, the same type of problem occurred. Thus, there are no existing optimization techniques that will allow the investigation of a city-sized routing problem. The development of heuristic methods has advanced, however, and feasible solutions for quite complex problems can be found quickly. No one has yet investigated the difference between a good feasible solution and an optimal solution in vehicle routing to determine if the search for optimal procedures is worthwhile. Meanwhile, sensitivity analysis using heuristic rather than optimal procedures must be viewed with extreme caution and care.

Perhaps the problems of routing would benefit from a shift in emphasis from discrete problems to continuous problems. Vehicle scheduling is viewed as a problem of finding routes between discrete points or in terms of a network example, in finding which arcs to travel to visit all nodes. The rapidly increasing combinatorial nature of the problem is what makes computation of optimal solutions so difficult. Studying public services in city street networks would reveal a continuous distribution of demand along an arc with a requirement that all arcs be traveled. For certain cases, this turns out to be a problem that is remarkably easy and quick to solve. The chinese postman problem asks how a continuous route for one vehicle can be found through a network that travels all arcs and minimizes total distance traveled. The extension of these techniques to problems of more than one service vehicle, each with a capacity constraint, while retaining the property of efficient solution would mean a dramatic increase in the type of analysis done on vehicle routing in solid waste collection.

The overall conclusion of this thesis is that models such as developed, limited though they may be by applying only to subproblems of the total collection system, can still provide a great deal of insight into the system. The types of questions amenable to study using these techniques, provide a wealth of information to the decision maker charged with the operation of the system. It is hoped that such models will become increasingly valuable tools in the management and planning of large-scale solid waste collection systems.

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